Calculus 2 — Exam 1
MAT 308, Spring 2020 — D. Ivanšić

Name: _____

Show all your work!

Find the following integrals:

1. (6pts) $\int x\cos(3x)\,dx =$

2. (9pts) $\int \sin^3 x\cos^3 x\,dx =$

Determine whether the following improper integrals converge, and, if so, evaluate them.

3. (8pts) $\int_1^\infty \frac{x+1}{x^3}\,dx =$

4. (8pts) $\int_0^\infty \frac{1}{1+x^2}\,dx =$

Use trigonometric substitution to evaluate the following integrals. Don't forget to return to the original variable where appropriate.

5. (12pts) $\displaystyle\int \frac{x^3}{\sqrt{x^2+7}}\,dx =$

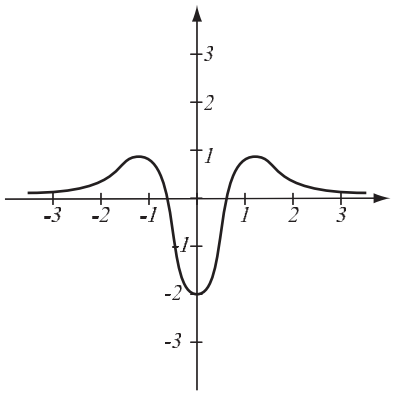
6. (14pts) $\displaystyle\int_0^{\sqrt{2}} x^2\sqrt{4-x^2}\,dx =$

Use the method of partial fractions to find the following integrals.

7. (14pts) $\displaystyle\int \frac{3x^3 - 5x^2 + 9x - 5}{(x^2+1)^2}\,dx =$

8. (9pts) Use comparison to determine whether the improper integral $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\cos x}{x}\,dx$ converges.

9. (20pts) The integral $\int_0^3 e^{-x^2}\,dx$ is given. It cannot be found by antidifferentiation, since the antiderivative of e^{-x^2} is not expressible using elementary functions.
a) Write the expression you would use to calculate M_6, the midpoint rule with 6 subintervals. All the terms need to be explicitly written, do not use f in the sum.
b) Find y'' for $y = e^{-x^2}$.
c) The graph of y'' is shown: use it to find the error estimate for M_n in general.
d) Estimate the error for M_6.
e) What should n be in order for M_n to give you an error less than 10^{-4}?



Bonus (10pts) Find the reduction formula that reduces $\displaystyle\int \frac{dx}{(x^2+a^2)^n}$ to $\displaystyle\int \frac{dx}{(x^2+a^2)^{n-1}}$. Start on $\displaystyle\int \frac{dx}{(x^2+a^2)^{n-1}}$ with an integration by parts, then rewrite an x^2 in the new integral as $x^2 + a^2 - a^2$ and see what you can do.