

Calculus 2 — Exam 0
MAT 308, Spring 2020 — D. Ivanšić

Name: _____
Show all your work!

Differentiate and simplify where appropriate:

1. (6pts) $\frac{d}{dx} \left(5x^4 - u^3 + \sqrt[4]{x^7} - \frac{12}{x^5} \right) =$

2. (6pts) $\frac{d}{dt} (\sqrt[3]{t} - 1)(\sqrt[3]{t}^2 + \sqrt[3]{t} + 1) =$

3. (8pts) $\frac{d}{dz} \frac{4z^2 + 1}{(z - 5)^2} =$

4. (4pts) $\frac{d}{dx} \frac{1}{xe^x} =$

5. (7pts) (This is a known derivative, your job is to verify it here.)

$$\frac{d}{d\theta} \ln |\sec \theta + \tan \theta| =$$

6. (6pts) $\frac{d}{dx} \left(\sqrt{x} - \frac{3}{\sqrt{x}} \right) \ln x =$

7. (5pts) Let $f(x) = 2^{7x}$. What is $f^{(44)}(x)$, the 44th derivative of f ? Justify your answer.

Find the following limits. Use L'Hospital's rule if needed.

8. (2pts) $\lim_{x \rightarrow \infty} \frac{1}{x^2} =$

9. (6pts) $\lim_{x \rightarrow \infty} \frac{x^3 - 5x^2 + 7x + 9}{x^2 - 4x + 5} =$

10. (8pts) $\lim_{x \rightarrow 0} (1 + 2x)^{\frac{1}{x}} =$

Find the following antiderivatives.

11. (7pts) $\int 4x^4 - \frac{4}{\sqrt{1-x^2}} + \sqrt[3]{x^{11}} + e^3 dx =$

12. (3pts) $\int \cos(4x + 1) dx =$

13. (7pts) $\int \frac{x^2 + 1}{\sqrt{x}} dx =$

Use the substitution rule in the following integrals:

14. (7pts) $\int \frac{4x - 3}{(4x^2 - 6x + 5)^3} dx =$

15. (10pts) $\int_0^{\frac{2\pi}{3}} \frac{\sin x}{1 + \cos^2 x} dx =$

16. (8pts) Find the equation of the tangent line to the curve $y = x^2 + 2x - 15$ at the point $(2, -7)$. Sketch the curve and the tangent line on the same graph.

Bonus. (10pts) The rear inside cover of our book claims that

$$\int x^2 \sqrt{a^2 - x^2} dx = \frac{x}{8}(2x^2 - a^2)\sqrt{a^2 - x^2} + \frac{a^4}{8} \arcsin \frac{x}{a} + C$$

Verify this formula by differentiating.