

College Algebra — Joysheet 1  
MAT 140, Fall 2020 — D. Ivanišić

Name: \_\_\_\_\_

Covers: JIT 6, 7, 12–14, 17      Show all  
your work!

Write interval notation and sketch on the number line.

1. (3pts)  $\{x|x \leq -2\}$

2. (3pts)  $\{x|0 < x \leq 5\}$

Solve the equations.

3. (3pts)  $3x - 7 = 5x + 2$

4. (4pts)  $4(2x - 4) - x = 5 - 3(x + 1)$

Simplify and write in standard form:

5. (4pts)  $(3x^2 + 4)(x - 2) - x^2(2x - 4) =$

6. (4pts)  $(2x - 3)(x^2 + 9x - 8) =$

Simplify and write the answer so all exponents are positive:

7. (2pts)  $(2u)^3u^4 =$

8. (2pts)  $\frac{x^7}{(3x)^4} =$

9. (3pts)  $(3s^3t^2)^3s^2t^4 =$

10. (5pts)  $(5x^2y^{-4})^2(x^3y^{-1})^3 =$

11. (7pts)  $\frac{(2u^4v^{-3})^4}{(10u^5v^{-2})^2} =$

Factor the following.

12. (4pts)  $x^2 + 5x - 14 =$

13. (4pts)  $x^2 - 20x - 44 =$

Solve the equations.

14. (6pts)  $x^2 - x = x + 8$

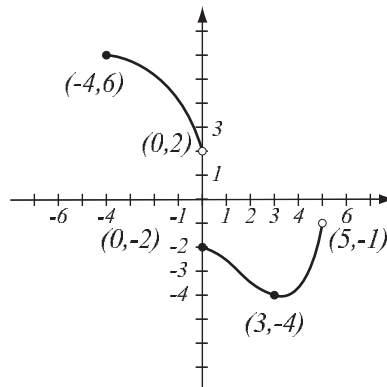
15. (6pts)  $2x^2 - 3x - 24 = x^2 + 2x$

1. (8pts) Let  $A = (-1, 2)$ ,  $B = (5, 4)$  and  $C = (8, -6)$ . Draw the triangle  $ABC$  and then determine algebraically if it is a right triangle.

2. (10pts) There are two circles that have radius 5, whose center is on the  $x$ -axis, and which contain the point  $(0, 4)$ . Find the equations of the two circles, and sketch them.

3. (8pts) Use the graph of the function  $f$  at right to answer the following questions.

- a) Find  $f(3)$  and  $f(0)$ .
- b) What is the domain of  $f$ ?
- c) What is the range of  $f$ ?
- d) What are the solutions of the equation  $f(x) = -3$ ?



4. (12pts) The function  $f(x) = x^2 - 8x - 5$  is given.

a) Use your calculator to accurately its graph. Draw the graph here, and indicate units on the axes.

b) Find all the  $x$ - and  $y$ -intercepts (accuracy: 6 decimal points).

c) State the domain and range.

5. (12pts) Find the domain of each function and write it using interval notation.

$$g(x) = \frac{\sqrt{x}}{x^2 + 4x - 5}$$

$$f(x) = \frac{\sqrt[3]{x}}{2x + 3}$$

6. (10pts) Let  $g(x) = \frac{\sqrt{x-4}}{x^2-9}$ . Find the following (simplify where appropriate).

$$g(1) =$$

$$g(6) =$$

$$g(-\sqrt{t}) =$$

$$g(x+5) =$$

1. (10pts) Find the equation of the line (in form  $y = mx + b$ ) passing through  $(-4, 3)$  and is parallel to the line  $3x + 2y = 8$ . Draw both lines.

2. (4pts) Find the equation of a line that has  $x$ -intercept 5 and slope 2. Draw the graph of the line.

3. (10pts) Draw the quadrangle with vertices  $A = (-1, 3)$ ,  $B = (1, 6)$ ,  $C = (3, -3)$  and  $D = (0, -1)$ .

a) Find the slopes of the sides of the quadrangle.

b) Use slopes to see if any two sides are parallel or perpendicular and identify which ones are (perpendicular sides need not meet at a vertex).

4. (4pts) According to the census bureau, the median household income in Kentucky was \$41,724 in 2012, and \$46,659 in 2016. What is the average rate of change of Kentucky's median household income from 2012 to 2016? What are the units for the average rate of change?

5. (12pts) In a big city, you twice used the same company for a cab ride. On the first occasion, you paid \$10.94 to ride 5 miles. On the second occasion, you paid \$22.77 to ride 12 miles.

a) Assuming that the cab ride cost  $C(x)$  is a linear function of the number of miles ridden  $x$ , write a formula for  $C(x)$ .

b) What is the cost if you traveled zero miles? What is the meaning of this number?

c) What is the meaning of the slope in this example?

6. (20pts) A statistician is trying to establish a relationship between McCracken county's population  $P$  and year  $t$ . In the table,  $P$  is McCracken county's population and  $t$  is the year. Solve the problems below with accuracy 6 decimal points.

a) Draw the scatterplot of the data. Does the relationship look linear?

b) Use two points in the scatterplot to get an equation of a line that models the relationship between  $P$  and  $t$ . Draw the line on the graph.

c) Use your calculator to find the "line of best fit" for the data. Draw the line on the graph.

d) Find the coefficient of correlation  $r$ . How strong is the linear relationship between  $P$  and  $t$ ?

e) What population can the statistician expect in year 2020?

$t$	$P$
1960	57,306
1970	58,281
1980	61,310
1990	62,879
2000	65,514
2010	65,565

Solve the inequalities. Write your solution in interval notation.

1. (5pts)  $8 \leq 3x - 1 < 11$

2. (7pts)  $3 + 2x < 4$  or  $4 + 3x > 2$

3. (6pts) Find the domain of the function in interval notation:  $f(x) = \frac{\sqrt{7 - 2x}}{x + 4}$ .

4. (14pts) Jay plans to invest \$8,000: part at 3.5% simple interest, and the rest at 4.75% simple interest. What is the most he can invest at 3.5% to guarantee receiving \$330 in interest in a year? Solve as an inequality.

**5.** (14pts) Amy, who rows at speed 7mph in still water, takes 3 hours to row upstream to a point on the river. Her return trip downstream takes 2 hours.

a) How fast does the river flow?

b) How far did Amy travel upstream?

**6.** (14pts) How many liters of a 3% solution of sulphuric acid must be mixed with 2 liters of a 7% solution of sulphuric acid in order to get a 6% solution of sulphuric acid?



1. (10pts) Use your calculator to accurately sketch the graph of the function

$f(x) = \frac{3|x|}{x^2 + 1}$ . Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.

a) Find the local maxima and minima for this function.

b) State the intervals where the function is increasing and where it is decreasing.

2. (20pts) Let  $f(x) = x^2 + 4x - 21$ ,  $g(x) = 3x - 2$ . Find the following (simplify where possible):

$$(f - g)(-1) =$$

$$(fg)(-2) =$$

$$\frac{f}{g}\left(\frac{2}{3}\right) =$$

$$(g \circ f)(3) =$$

$$(f \circ g)(x) =$$

The domain of  $\frac{g}{f}(x)$  in interval notation

3. (8pts) Consider the function  $h(x) = \sqrt{7x - 1}$  and find **two** different solutions to the following problem: find functions  $f$  and  $g$  so that  $h(x) = f(g(x))$ , where neither  $f$  nor  $g$  are the identity function.

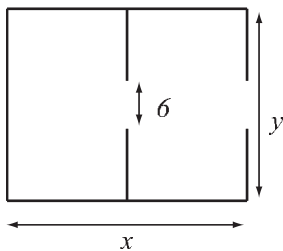
4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} -x - 2, & \text{if } x < 1 \\ -2x + 4, & \text{if } 1 \leq x < 5. \end{cases}$$

5. (14pts) Gloria is building a two-room store with area 1500 square feet and 6-foot openings for doors. She wishes to minimize the building cost, which is the same as minimizing the total length of the walls.

a) Express the total length of the walls of the building as a function of the length of one of the sides  $x$ . What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the store for which the total length of the walls is minimal? What is the minimal wall length?



1. (21pts) For the following functions:

a) determine algebraically whether they are odd, even, or neither

b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$$f(x) = x^3 - |x|$$

$$g(x) = -x^2 - 7x + 5$$

$$h(x) = x^3 + 5x$$

2. (16pts) Draw the graphs of  $f(x) = \frac{1}{2x} - 4$  and  $g(x) = -3|x - 2|$  using transformations. Explain how you transform graphs of basic functions in order to get the graphs of  $f$  and  $g$ . Indicate at least two points on each graph.

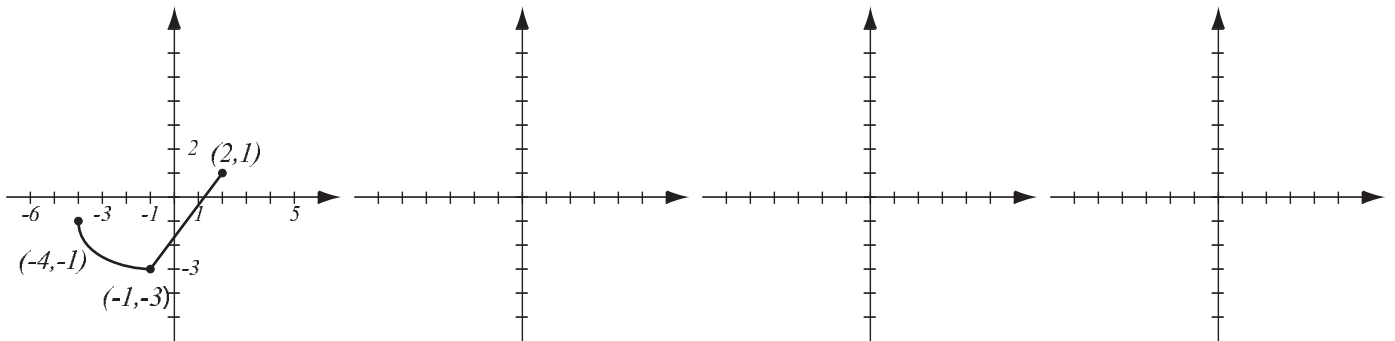
3. (10pts) Write the equation for the function whose graph has the following characteristics:

a) shape of  $y = x^3$ , shifted down 2 units

b) shape of  $y = \sqrt{x}$  shifted left 3 units, then stretched vertically by factor 2

c) shape of  $y = 2x + 1$ , stretched horizontally by factor 4, then reflected about the  $y$ -axis, then reflected about the  $x$ -axis.

4. (13pts) The graph of  $f(x)$  is drawn below. On three separate graphs, sketch the graphs of the functions  $f(x - 2)$ ,  $f(-x) + 3$  and  $-f(2x)$  and label all the relevant points.



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*Covers: JIT 13, 15, 21–23 Show all your work!*

Use formulas to expand:

1. (4pts)  $(u + 7)^2 =$

2. (5pts)  $(x^2 + 3y)^2 =$

3. (4pts)  $(5x + 7y)(5x - 7y) =$

Use the *ac*-method or another method to factor. Show how you got your answer.

4. (5pts)  $7x^2 - 19x - 6 =$

5. (6pts)  $12x^2 + x - 6 =$

6. (8pts) Compute expressions with fractions by hand.

$$\frac{3}{7} \cdot \frac{4}{9} =$$

$$\frac{15}{4} \div \frac{9}{10} =$$

$$\frac{7}{6} + \frac{3}{10} =$$

$$\frac{5}{12} - \frac{11}{42} =$$

Multiply or divide the rational expressions.

7. (7pts)  $\frac{6x + 3}{2x^2 - 9x - 5} \cdot \frac{x^2 - 25}{3x + 12} =$

8. (7pts)  $\frac{x^2 - 16}{3x^2 + 5x - 12} \div \frac{x^2 + 7x + 12}{6x^2 - 8x} =$

Add or subtract the rational expressions.

9. (6pts)  $\frac{x + 2}{x^2 - 2x - 3} - \frac{5x}{x + 1} =$

10. (8pts)  $\frac{x}{2x^2 - 3x - 35} + \frac{x - 2}{x^2 - 6x + 5} =$

College Algebra — Joysheet 8  
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*Covers: 3.1, 3.2, 3.3 Show all your work!*

Simplify, so that the answer is in form  $a + bi$ .

1. (4pts)  $(1 + i)(3 - i)^2 =$

2. (6pts)  $\frac{-2 + i}{4 - 5i} =$

3. (4pts) Simplify and justify your answer.

$i^{271} =$

4. (8pts) The number of crates of grapes in storage of a grape grower is described by the function  $S(x) = -x^2 + 26x + 15$ , where  $x$  is the number of days after September 25th.

a) On what dates did the grower have 135 crates in storage?

b) On what date did the number of crates in storage peak?

5. (8pts) Solve the equation:  $x^4 + x^2 - 20 = 0$

6. (6pts) Solve by completing the square.

$x^2 + 8x - 1 = 0$

7. (12pts) The quadratic function  $f(x) = \frac{1}{2}x^2 + 4x + 6$  is given. Do the following without using the calculator.

- a) Find the  $x$ -intercepts of its graph, if any. Find the  $y$ -intercept.
- b) Find the vertex of the graph.
- c) Sketch the graph of the function.

8. (12pts) Jen is a campaign manager for a political campaign. Her base hourly pay of \$13 is increased by a bonus of \$0.25 for every volunteer she recruits. However, having more volunteers reduces her workload, so her 40 weekly hours are reduced by a half hour for every volunteer she recruits. One week, Jen was paid \$544. How many volunteers did she recruit that week?



College Algebra — Joysheet 9  
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Name: \_\_\_\_\_

*Covers: 3.3, 3.4, 3.5 Show all your work!*

1. (4pts) Solve the equation.

$$|4x - 5| = 7$$

2. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$$|x + 7| \leq 3$$

$$|2x - 6| < 9$$

Solve the equations:

3. (8pts)  $\frac{x + 3}{x + 1} + \frac{8x}{x^2 - 6x - 7} = \frac{7}{x - 7}$

4. (8pts)  $x + \sqrt{40 - 3x} = 4$

5. (14pts) A lemon is launched from height 11 feet upwards with initial velocity 56 feet per second. Its height in feet after  $t$  seconds is given by  $s(t) = -16t^2 + 56t + 11$ .

a) Sketch the graph of the height function.

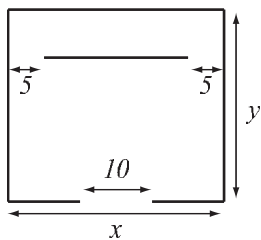
b) When does the lemon reach its greatest height, and what is that height?

c) When is the lemon at height 44 feet?

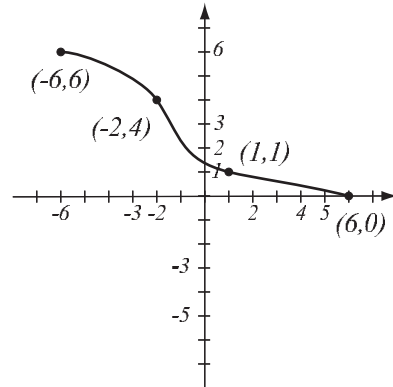
6. (14pts) Matthew is building a gas station convenience store with the floor plan below. Openings of 5 feet are left for restroom doors and another opening of 10 feet is left for the entrance door. Matthew has budgeted for 500 ft of walls and wishes to maximize the area of the store.

a) Express the total area of the store as a function of one of the sides of the rectangle. What is the domain of this function?

b) Sketch the graph of the area function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the store that has the greatest total area? What is the greatest total area possible?



1. (6pts) The graph of a function  $f$  is given.
- Is this function one-to-one? Justify.
  - If the function is one-to-one, find the graph of  $f^{-1}$ , labeling the relevant points.



2. (12pts) Let  $f(x) = \frac{3x + 5}{4x - 1}$ . Find the formula for  $f^{-1}$ . Find the ranges of  $f$  and  $f^{-1}$ .

3. (8pts) Evaluate without using the calculator:

$$\log_9 81 =$$

$$\log_6 \frac{1}{216} =$$

$$\log_{100} 1000 =$$

$$\log_{a^3} a^{12} =$$

4. (4pts) Use the change-of-base formula and your calculator to find  $\log_4 30$  with accuracy 6 decimal places. Show how you obtained your number.

5. (12pts) Investigate the effect of increased frequency of compounding: for a deposit of \$4,000 and annual interest rate of 4.32%, calculate the amount in the account after 1 year for the frequencies of compounding below.

- Write the general formula for the amount, replacing the variables by numbers, if known.
- Use the table feature on your calculator to quickly compute amounts after 1 year.
- Does compounding more often make a big difference?

Frequency: every	$n$	Amount after 1 year
year		
quarter		
month		
day		
hour		
second		

6. (3pts) Find the domain of  $f(x) = \ln(2x - 7)$ .

7. (8pts) The cost of a cab ride in some town is given by  $C(x) = 1.75 + 1.45x$ , where  $C$  is in dollars and  $x$  is the distance driven in miles.

- Find the cost of a 5- and 13-mile ride.
- Find a formula for the inverse function and explain what it represents.
- Riding in this cab, how far can you get on \$10? \$20?

8. (7pts) Using transformations, draw the graph of  $f(x) = -e^{x-5}$ . Explain how you transform the graph of a basic function in order to get the graph of  $f$ . Show at least one point on the graph, and asymptotes to the graph, if any.

1. (5pts) If  $\log_a 3 = u$  and  $\log_a 8 = v$ , express in terms of  $u$  and  $v$ :

$$\log_a 72 = \qquad \qquad \qquad \log_a \frac{8}{3} =$$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\ln(e^3 x^8 y^{-2}) =$$

$$\log_8 \frac{x^{-\frac{8}{3}} y^2}{64 \sqrt[3]{xy^4}} =$$

3. (12pts) Write as a single logarithm. Simplify if possible.

$$2 \log(5x^3) + \frac{1}{2} \log(625y^4) - 5 \log x =$$

$$\log_2(x - 4) + 3 \log_2(x + 4) - 2 \log_2(x^2 - 16) =$$

Solve the equations.

4. (5pts)  $16^{2x-1} = \left(\frac{1}{2}\right)^{3x+5}$

5. (7pts)  $4^{x-1} = e^{2x+3}$

6. (8pts)  $\log_3(x + 14) + \log_3(x - 10) = 4$

7. (12pts) The town of Rabbiton had 23,000 inhabitants in 2015 and 27,000 in 2018. Assume the population of Rabbiton grows exponentially.

a) Write the function describing the number  $P(t)$  of people in Rabbiton  $t$  years after 2015. Then find the exponential growth rate for this population.

b) Graph the function.

c) According to this model, when will the population reach 40,000?