College Algebra — Joysheet 11
MAT 140, Fall 2020 — D. Ivanšić

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Covers: 5.4–5.6 *Show all your work!*

1. (5pts) If $\log_a 3 = u$ and $\log_a 8 = v$, express in terms of u and v:

$\log_a 72 = \log_a(3^2 \cdot 8)$

$= 9 \cdot 8$
$= 3^2 \cdot 8$

$= \log_a 3^2 + \log_a 8$

$= 2\log_a 3 + \log_a 8$

$= 2u + v$

$\log_a \frac{8}{3} = \log_a 8 - \log_a 3$

$= v - u$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\ln(e^3x^8y^{-2}) = \ln e^3 + \ln x^8 + \ln y^{-2}$

$= 3 + 8\ln x - 2\ln y$

$\log_8 \frac{x^{-\frac{8}{3}}y^2}{64\sqrt[3]{x}y^4} = \log_8 x^{-\frac{8}{3}} + \log_8 y^2 - \log_8 64 - \log_8 x^{\frac{1}{3}} - \log_8 y^4$

$= -\frac{8}{3}\log_8 x + 2\log_8 y - 2 - \frac{1}{3}\log_8 x - 4\log_8 y$

$= -3\log_8 x - 2\log_8 y - 2$

$-\frac{8}{3} - \frac{1}{3}$

3. (12pts) Write as a single logarithm. Simplify if possible.

$2\log(5x^3) + \frac{1}{2}\log(625y^4) - 5\log x = \log(5x^3)^2 + \log(625y^4)^{\frac{1}{2}} - \log x^5$

$\sqrt{625}$

$= \log \frac{(5x^3)^2 \cdot (625y^4)^{\frac{1}{2}}}{x^5} = \log \frac{25x^6 \cdot 25y^2}{x^5} = \log(625xy^2)$

$\log_2(x-4) + 3\log_2(x+4) - 2\log_2(x^2-16) = \log_2(x-4) + \log_2(x+4)^3 - \log_2(\underbrace{x^2-16}_{(x-4)(x+4)})^2$

$= \log_2 \frac{(x-4)(x+4)^3}{((x-4)(x+4))^2} = \log_2 \frac{(x-4)(x+4)^3}{(x-4)^2(x+4)^2}$

$= \log_2 \frac{x+4}{x-4}$

Solve the equations.

4. (5pts) $16^{2x-1} = \left(\frac{1}{2}\right)^{3x+5}$

$(2^4)^{2x-1} = (2^{-1})^{3x+5}$

$2^{8x-4} = 2^{-3x-5}$

$8x-4 = -3x-5$

$11x = -1 \quad x = -\frac{1}{11}$

5. (7pts) $4^{x-1} = e^{2x+3}$ $\mid \ln$

$\ln 4^{x-1} = \ln e^{2x+3}$

$(x-1)\ln 4 = 2x+3$

$x\ln 4 - \ln 4 = 2x+3$

$\ln 4 \cdot x - 2x = 3 + \ln 4$

$(\ln 4 - 2)x = 3 + \ln 4$

$x = \frac{3+\ln 4}{\ln 4 - 2} = -7.147228$

6. (8pts) $\log_3(x+14) + \log_3(x-10) = 4$

$\log_3((x+14)(x-10)) = 4$

$3^{\log_3((x+14)(x-10))} = 3^4$

$(x+14)(x-10) = 81$

$x^2 + 4x - 140 = 81$

$x^2 + 4x - 221 = 0$

$x = \frac{-4 \pm \sqrt{\overset{16+884}{4^2 - 4\cdot(-221)}}}{2} = \frac{-4 \pm \sqrt{900}}{2}$

$= \frac{-4 \pm 30}{2} = -\frac{34}{2}, \frac{26}{2} = -17, 13$

7. (12pts) The town of Rabbiton had 23,000 inhabitants in 2015 and 27,000 in 2018. Assume the population of Rabbiton grows exponentially.
a) Write the function describing the number $P(t)$ of people in Rabbiton t years after 2015. Then find the exponential growth rate for this population.
b) Graph the function.
c) According to this model, when will the population reach 40,000?

a) In thousands,

$P(t) = 23 \cdot e^{k \cdot t}$

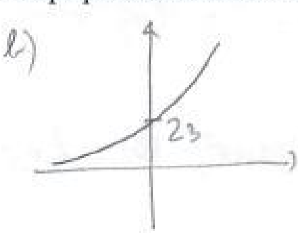
$27 = P(3) = 23 e^{k\cdot 3}$

$\frac{27}{23} = e^{k\cdot 3}$ $\mid \ln$

$\ln\frac{27}{23} = k\cdot 3$

$k = \frac{\ln\frac{27}{23}}{3} = 0.0534476$

b)



c) $P(t) = 40$

$23 \cdot e^{0.053..t} = 40$

$e^{0.053..t} = \frac{40}{23}$ $\mid \ln$

$0.053..t = \ln\frac{40}{23}$

$t = \frac{\ln\frac{40}{23}}{0.053..} = 10.3538$

10 years from 2015, in 2025, Rabbiton reaches 40,000.