College Algebra — Joysheet 10
MAT 140, Fall 2020 — D. Ivanšić

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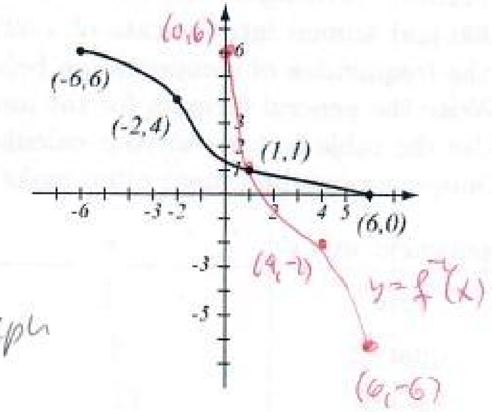
Covers: 5.1–5.3 *Show all your work!*

1. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.



a) Yes, it passes the horizontal line test.

b) See graph

2. (12pts) Let $f(x) = \dfrac{3x+5}{4x-1}$. Find the formula for f^{-1}. Find the ranges of f and f^{-1}.

$y = \dfrac{3x+5}{4x-1}$

$(4x-1)y = 3x+5$

$4xy - y = 3x+5$

$4xy - 3x = y+5$

$(4y-3)x = y+5$

$x = \dfrac{y+5}{4y-3} = f^{-1}(y)$

range of f = domain of f^{-1}: $(-\infty, \frac{3}{4}) \cup (\frac{3}{4}, \infty)$

can't have $4y-3=0$

$4y=3$

$y=\frac{3}{4}$

range of f^{-1} = domain of f = $(-\infty, \frac{1}{4}) \cup (\frac{1}{4}, \infty)$

can't have $4x-1=0$

$4x=1$

$x=\frac{1}{4}$

3. (8pts) Evaluate without using the calculator:

$\log_9 81 = 2$ $\quad 9^{?} = 81$

$\log_6 \frac{1}{216} = -3$ $\quad 6^{?} = \frac{1}{216} = \frac{1}{6^3} = 6^{-3}$

$\log_{100} 1000 = \frac{3}{2}$ $\quad 100^{?} = 1000 = 10^3 = (\sqrt{100})^3 = (100^{\frac{1}{2}})^3 = 100^{\frac{3}{2}}$

$\log_{a^3} a^{12} = 4$ $\quad (a^3)^{?} = a^{12}$

4. (4pts) Use the change-of-base formula and your calculator to find $\log_4 30$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_4 30 = \dfrac{\ln 30}{\ln 4} = 2.453445$

5. (12pts) Investigate the effect of increased frequency of compounding: for a deposit of \$4,000 and annual interest rate of 4.32%, calculate the amount in the account after 1 year for the frequencies of compounding below.
a) Write the general formula for the amount, replacing the variables by numbers, if known.
b) Use the table feature on your calculator to quickly compute amounts after 1 year.
c) Does compounding more often make a big difference?

$A = 4000\left(1 + \frac{0.0432}{n}\right)^{n}$ ← n·1

Frequency: every	n	Amount after 1 year
year	1	4172.80
quarter	4	4175.62
month	12	4176.26
day	365	4176.58
hour	$365 \cdot 24 = 8760$	4176.59
second	$365 \cdot 24 \cdot 3600 = 31,536,000$	4176.59

c) It makes some difference going from once a year to monthly, but after that the effect is quite small.

6. (3pts) Find the domain of $f(x) = \ln(2x - 7)$.

Must have $2x - 7 > 0$

$2x > 7$

$x > \frac{7}{2}$

$\left(\frac{7}{2}, \infty\right)$

7. (8pts) The cost of a cab ride in some town is given by $C(x) = 1.75 + 1.45x$, where C is in dollars and x is the distance driven in miles.
a) Find the cost of a 5- and 13-mile ride.
b) Find a formula for the inverse function and explain what it represents.
c) Riding in this cab, how far can you get on \$10? \$20?

a) $C(5) = 1.75 + 1.45 \cdot 5 = \$9$

$C(13) = 1.75 + 1.45 \cdot 13 = \$20.60$

b) $C = 1.75 + 1.45x$

$C - 1.75 = 1.45x$

$x = \frac{C - 1.75}{1.45}$ ← given an amount of money C, tells us how far we can ride with C dollars

c) $x = \frac{10 - 1.75}{1.45} = 5.689655$ miles

$x = \frac{20 - 1.75}{1.45} = 12.586206$ miles

8. (7pts) Using transformations, draw the graph of $f(x) = -e^{x-5}$. Explain how you transform the graph of a basic function in order to get the graph of f. Show at least one point on the graph, and asymptotes to the graph, if any.