College Algebra — Joysheet 8
MAT 140, Fall 2020 — D. Ivanšić

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Covers: 3.1, 3.2, 3.3 Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $(1+i)(3-i)^2 = (1+i)(3^2 - 2\cdot 3\cdot i + i^2)$

$= (1+i)(9-6i-1) = (1+i)(8-6i) = 8+8i-6i-6i^2$ ($i^2 = -1$)

$= 14+2i$

2. (6pts) $\frac{-2+i}{4-5i} = \frac{-2+i}{4-5i}\cdot\frac{4+5i}{4+5i} = \frac{-8+4i-10i+5i^2}{4^2-(5i)^2}$

$= \frac{-8-6i-5}{16-25i^2} = \frac{-13-6i}{41} = -\frac{13}{41} - \frac{6}{41}i$

3. (4pts) Simplify and justify your answer.

$i^{271} = i^{268}\cdot i^3 = i^{4\cdot 67}\cdot i^3 = (i^4)^{67}\cdot i^3 = i\cdot i\cdot i = -i$

$271 = 268 + 3 = 4\cdot 67 + 3$ ($i^4 = 1$)

4. (8pts) The number of crates of grapes in storage of a grape grower is described by the function $S(x) = -x^2 + 26x + 15$, where x is the number of days after September 25th.

a) On what dates did the grower have 135 crates in storage?

b) On what date did the number of crates in storage peak?

a) $-x^2 + 26x + 15 = 135$ $x = 6$ or $x = 20$

$-x^2 + 26x - 120 = 0$ Oct 1st (6 days after 9/25) Oct. 15th (20 days after 9/25)

$x^2 - 26x + 120 = 0$

$(x-6)(x-20) = 0$

b) $x = -\frac{b}{2a} = -\frac{26}{2\cdot(-1)} = 13$

Oct. 8th.

5. (8pts) Solve the equation: $x^4 + x^2 - 20 = 0$

$(x^2)^2 + x^2 - 20 = 0$ $u = -5, 4$

Let $u = x^2$ $u^2 + u - 20 = 0$ $x^2 = -5$ or $x^2 = 4$

$(u+5)(u-4) = 0$ $x = \pm\sqrt{5}i$ $x = \pm 2$

6. (6pts) Solve by completing the square.

$x^2 + 8x - 1 = 0$ $\quad | + 4^2$ $\quad (x+4)^2 = 17$

$x^2 + 2\cdot x\cdot 4 + 4^2 - 1 = 4^2$ $\quad x + 4 = \pm\sqrt{17}$

$(x+4)^2 = 16 + 1$ $\quad x = -4 \pm \sqrt{17}$

7. (12pts) The quadratic function $f(x) = \frac{1}{2}x^2 + 4x + 6$ is given. Do the following without using the calculator.

a) Find the x-intercepts of its graph, if any. Find the y-intercept.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

a) $\frac{1}{2}x^2 + 4x + 6 = 0 \quad | \cdot 2$

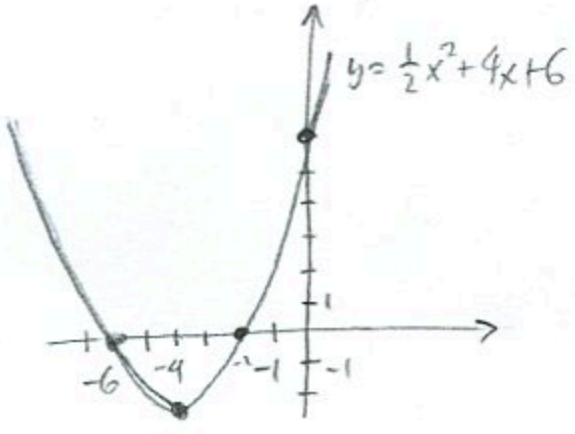
$x^2 + 8x + 12 = 0$

$(x+6)(x+2) = 0$

$x = -6, -2$ x-int.

y-int: $f(0) = 6$

b) $h = -\frac{b}{2a} = -\frac{4}{2 \cdot (\frac{1}{2})} = -4$



$f(-4) = \frac{1}{2}(-4)^2 + 4(-4) + 6 = 8 - 16 + 6 = -2$ Vertex: $(-4, -2)$

8. (12pts) Jen is a campaign manager for a political campaign. Her base hourly pay of \$13 is increased by a bonus of \$0.25 for every volunteer she recruits. However, having more volunteers reduces her workload, so her 40 weekly hours are reduced by a half hour for every volunteer she recruits. One week, Jen was paid \$544. How many volunteers did she recruit that week?

x = no of volunteers recruited

Jen's hourly pay is $13 + 0.25x = 13 + \frac{1}{4}x$

Jen's hours are $40 - \frac{1}{2} \cdot x$

Weekly pay = hourly pay · hours worked

$544 = (13 + \frac{1}{4}x) \cdot (40 - \frac{1}{2}x)$

$520 + 10x - \frac{13}{2}x - \frac{1}{8}x^2 = 544 \quad | -544$

$-\frac{1}{8}x^2 + \frac{2}{2}x - 24 = 0 \quad | (-8)$

$x^2 - 28x + 192 = 0$

$x = \frac{-(-28) \pm \sqrt{(-28)^2 - 4 \cdot 1 \cdot 192}}{2 \cdot 1}$

$= \frac{28 \pm \sqrt{784 - 768}}{2}$

$= \frac{28 \pm \sqrt{16}}{2} = \frac{28 \pm 4}{2} = 16, 12$

Jen hired either 16 or 12 volunteers.