1. (21pts) For the following functions:
a) determine algebraically whether they are odd, even, or neither
b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$f(x) = x^3 - |x|$

$f(-x) = (-x)^3 - |-x|$

$= -x^3 - |x|$

$\neq f(x)$ neither
$\neq -f(x)$

$g(x) = -x^2 - 7x + 5$

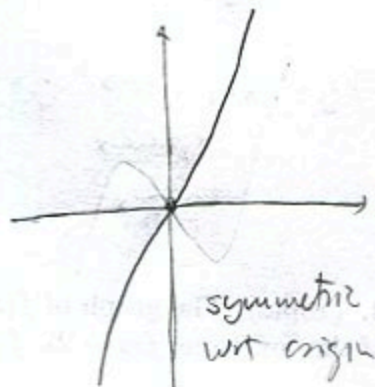$g(-x) = -(-x)^2 - 7(-x) + 5$

$= -x^2 + 7x + 5$

$\neq g(x)$ neither
$\neq -g(x)$

$h(x) = x^3 + 5x$
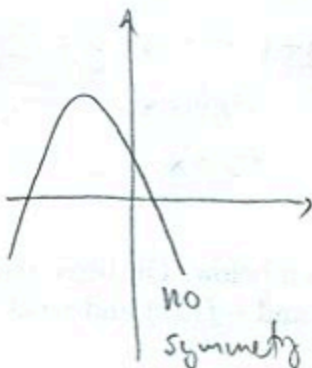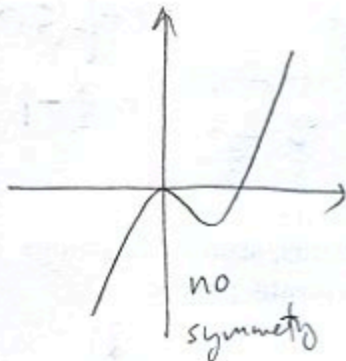
$h(-x) = (-x)^3 + 5(-x)$

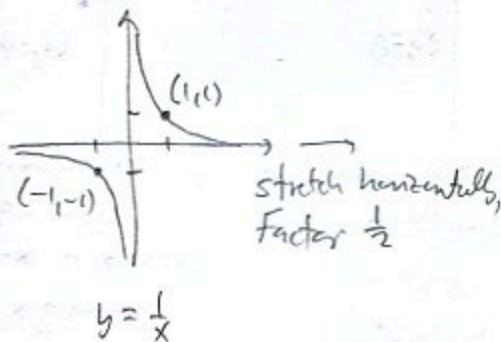$= -x^3 - 5x$

$= -h(x)$

odd

no symmetry

no symmetry

symmetric wrt origin
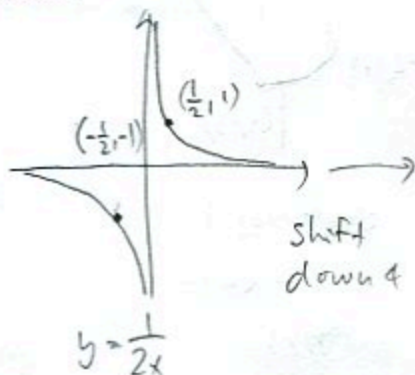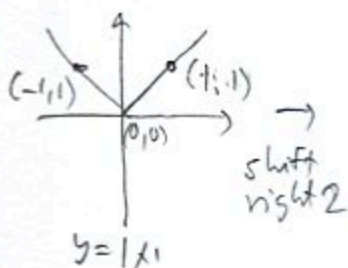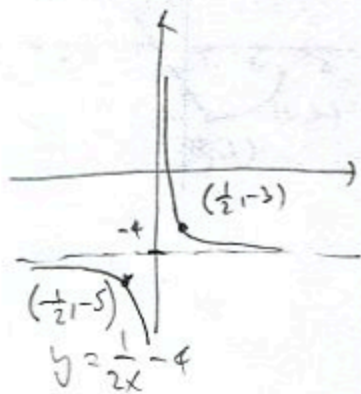
2. (16pts) Draw the graphs of $f(x) = \dfrac{1}{2x} - 4$ and $g(x) = -3|x - 2|$ using transformations. Explain how you transform graphs of basic functions in order to get the graphs of f and g. Indicate at least two points on each graph.
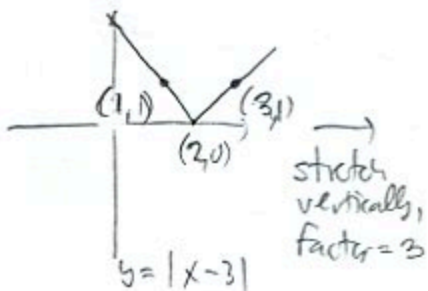

$(1,1)$
$(-1,-1)$
$y = \frac{1}{x}$

stretch horizontally, factor $\frac{1}{2}$

$(\frac{1}{2}, 1)$
$(-\frac{1}{2}, -1)$
$y = \frac{1}{2x}$

shift down 4

$(\frac{1}{2}, -3)$
$(-\frac{1}{2}, -5)$
$y = \frac{1}{2x} - 4$

$(-1,1)$ $(1,1)$
$(0,0)$
$y = |x|$

shift right 2

$(1,1)$ $(3,1)$
$(2,0)$
$y = |x - 3|$

stretch vertically, factor = 3

$(1,3)$ $(3,3)$
$(2,0)$
$y = 3|x - 2|$

reflect in x-axis

$(2,0)$
$(1,-3)$ $(3,-3)$
$y = -3|x - 2|$
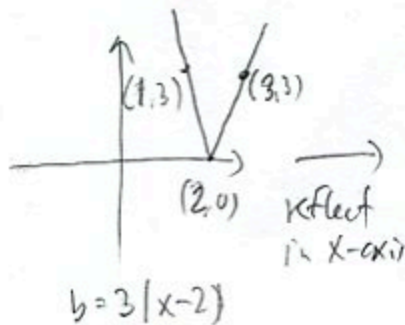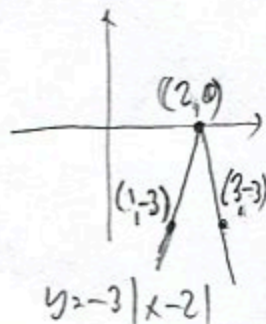
**3.** (10pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = x^3$, shifted down 2 units
b) shape of $y = \sqrt{x}$ shifted left 3 units, then stretched vertically by factor 2
c) shape of $y = 2x + 1$, stretched horizontally by factor 4, then reflected about the y-axis, then reflected about the x-axis.
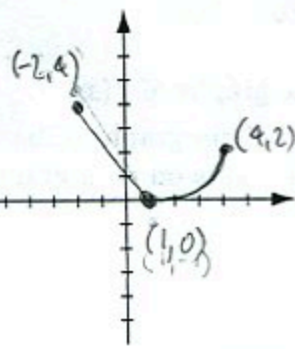
a) $y = x^3 \rightsquigarrow y = x^3 - 2$
   subtract 2

b) $y = \sqrt{x} \rightsquigarrow y = \sqrt{x+3} \longrightarrow y = 2\sqrt{x+3}$
   replace   multiply eq.
   x by $x+3$ by 2

c) $y = 2x + 1 \longrightarrow y = 2 \cdot (\frac{1}{4}x) + 1 \longrightarrow y = 2(\frac{1}{4}(-x)) + 1 \rightsquigarrow y = -\left[2\left(\frac{1}{4}(-x)\right)+1\right]$
   replace x replace x multiply
   by $\frac{1}{4}x$ by $-x$ eq. by -1 $= \frac{x}{2} - 1$

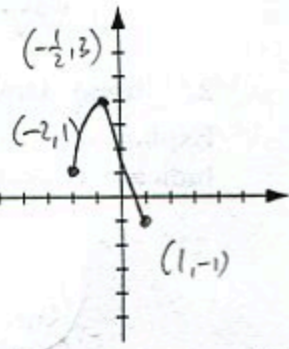**4.** (13pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x-2)$, $f(-x)+3$ and $-f(2x)$ and label all the relevant points.

(2,1)   (-4,-1)   (-1,-3)

(4,1)   (-2,-1)   (1,-3)
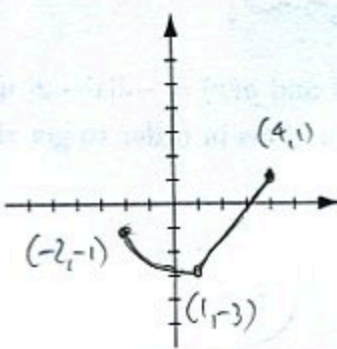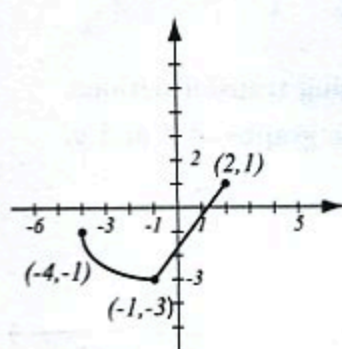
shift right 2
$x \mapsto x+2$
$y \mapsto y$

(-2,4)   (4,2)   (1,0)

reflect in y-axis
shift up 3
$x \mapsto -x$
$y \mapsto y+3$

(-½,3)   (-2,1)   (1,-1)

stretch horizontally,
factor $= \frac{1}{2}$
reflect in x-axis
$x \mapsto \frac{1}{2}x$
$y \mapsto -y$