College Algebra — Joysheet 5 MAT 140, Fall 2020 — D. Ivanšić

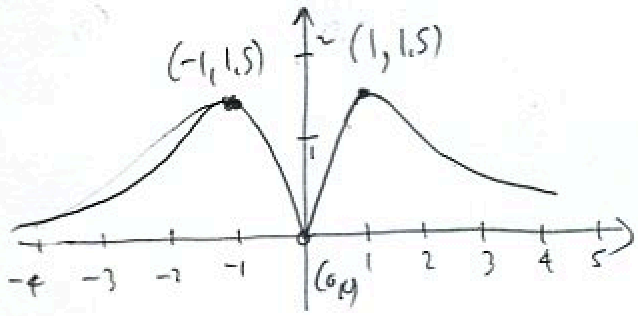
Name: Saul Ocean

Covers: 2.1, 2.2, 2.3 Show all your work!

1. (10pts) Use your calculator to accurately sketch the graph of the function $f(x) = \dfrac{3|x|}{x^2+1}$. Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.

a) Find the local maxima and minima for this function.

b) State the intervals where the function is increasing and where it is decreasing.



a) $f(1)=1.5$ is a local maximum
$f(-1)=1.5$ is a local maximum
$f(0)=0$ is a local minimum

b) Increasing on $(-\infty,-1)$ and $(0,1)$
Decreasing on $(-1,0)$ and $(0,\infty)$

2. (20pts) Let $f(x) = x^2 + 4x - 21$, $g(x) = 3x - 2$. Find the following (simplify where possible):

$(f-g)(-1) = f(-1)-g(-1) = ((-1)^2+4(-1)-21)-(3\cdot(-1)-2)$
$=(1-4-21)-(-3-2) = -24+5 = -19$

$(fg)(-2) = f(-2)\cdot g(-2) = (4-8-21)\cdot(-6-2)$
$= -25\cdot(-8) = 200$

$\dfrac{f}{g}\left(\tfrac{2}{3}\right) = \dfrac{f(\frac{2}{3})}{g(\frac{2}{3})} = \dfrac{(\frac{2}{3})^2+4\cdot\frac{2}{3}-21}{3\cdot\frac{2}{3}-2}$
$= \dfrac{\frac{4}{3}+\frac{8}{3}-21}{0}$ = not defined

$(g\circ f)(3) = g(f(3)) = g(3^2+4\cdot3-21)$
$= g(0) = 3\cdot0-2 = -2$

$(f\circ g)(x) = f(g(x)) = f(3x-2) = (3x-2)^2+4\cdot(3x-2)-21$
$= 9x^2-2\cdot3x\cdot2+4+12x-8-21$
$= 9x^2-25$

The domain of $\dfrac{g}{f}(x)$ in interval notation

Domain of g = all real numbers
Domain of f = all real numbers

overlap is all real numbers, exclude when $f(x)=0$

$x^2+4x-21=0$
$(x+7)(x-3)=0$
$x=-7, 3$

-7 3

$(-\infty,-7)\cup(-7,3)\cup(3,\infty)$

3. (8pts) Consider the function $h(x) = \sqrt{7x-1}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

$g(x) = 7x-1 \qquad g(x) = 7x$

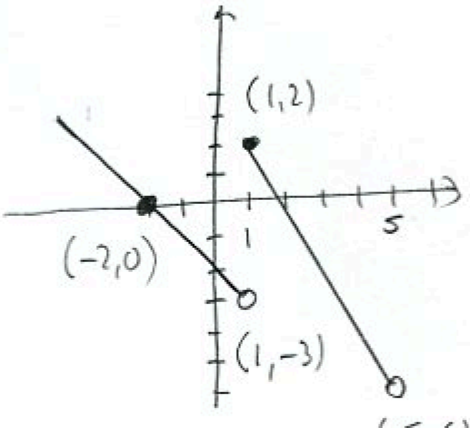
$f(x) = \sqrt{x} \qquad f(x) = \sqrt{x-1}$

4. (8pts) Sketch the graph of the piecewise-defined function:

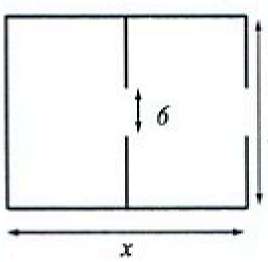
$$f(x) = \begin{cases} -x-2, & \text{if } x < 1 \\ -2x+4, & \text{if } 1 \le x < 5. \end{cases}$$

x	$-x-2$
-2	0
1	-3

x	$-2x+4$
1	2
5	-6



5. (14pts) Gloria is building a two-room store with area 1500 square feet and 6-foot openings for doors. She wishes to minimize the building cost, which is the same as minimizing the total length of the walls.
a) Express the total length of the walls of the building as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the store for which the total length of the walls is minimal? What is the minimal wall length?



a) $\ell = 2x + y + 2(y-6)$

$= 2x + 3y - 12$

$1500 = xy$, so $y = \frac{1500}{x}$

$\ell = 2x + 3 \cdot \frac{1500}{x} - 12$

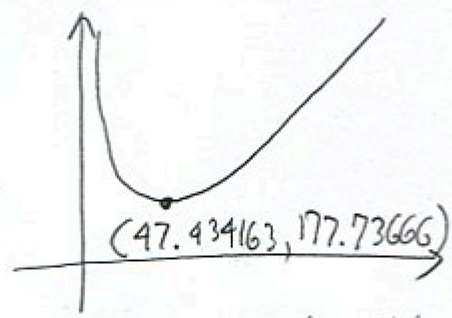
$\ell(x) = 2x + \frac{4500}{x} - 12$

Domain:
Must have
$x > 0$
$y \geq 6$
$\frac{1500}{x} \geq 6$
$1500 \geq 6x$

$x \leq \frac{1500}{6} = 250$

$(0, 250]$ - domain

b)



Dimensions for minimal wall length:
$x \times y = 47.434163 \times 31.622778$ (ft)

Minimal wall length:
$\ell = 177.73666$ ft