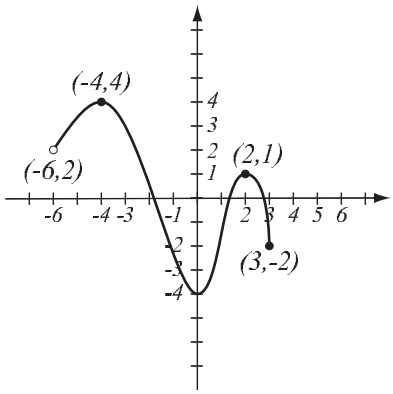
College Algebra — Exam 1
MAT 140, Fall 2020 — D. Ivanšić

Name: _____

Show all your work!

1. (8pts) Use the graph of the function f at right to answer the following questions.



a) Find: $f(2) =$ $f(-6) =$

b) What is the domain of f?

c) What is the range of f?

d) What are the solutions of the equation $f(x) = 3$?

2. (10pts) Use your calculator to accurately sketch the graph of $y = -x^3 + 8x + 12$.

a) Draw the graph on paper and indicate units on the axes.

b) Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).

3. (5pts) Write the equation of the line whose x-intercept is 3, and y-intercept is -5.

4. (10pts) Find the equation of the line (in form $y = mx + b$) that is perpendicular to the line $x + 4y = -8$ and passes through $(2, -1)$. Draw both lines.

5. (8pts) Draw the triangle with vertices $A = (0, 0)$, $B = (3, 2)$ and $C = (7, -4)$ in the coordinate plane. Use the Pythagorean theorem to determine if the triangle is a right triangle.

6. (9pts) Let $f(x) = x^2 + 4x - \dfrac{1}{\sqrt{x+1}}$. Find the following (simplify where appropriate).

$f(3) =$ $f(-6) =$

$f(\sqrt{u}) =$ $f(t-3) =$

7. (9pts) Find the domains of the functions below and write them using interval notation.

$f(x) = \dfrac{x-3}{2x-5}$ $g(x) = \dfrac{\sqrt{5-3x}}{x+8}$

8. (5pts) Solve and write the solution in interval notation.

$-2 \leq 5 - 2x < 4$

9. (10pts) The diameter of a circle has endpoints $(-3, 2)$ and $(1, 0)$.
a) Find the equation of the circle.
b) Draw the circle in the coordinate plane.

10. (12pts) Ellen plans to invest \$12,000: part at 3.5% simple interest, and the rest at 4.5% simple interest. What is the most she can invest at 3.5% to guarantee receiving \$500 in interest in a year? Solve as an inequality.

11. (14pts) Amy and Mitch bicycle along the same road. It takes Mitch 1 hour to travel the road. Amy leaves 12 minutes after Mitch, but gets to the end of the road at the same time as Mitch because she travels 2 mph faster than him.
a) What are the speeds of the cyclists?
b) How long is the road?

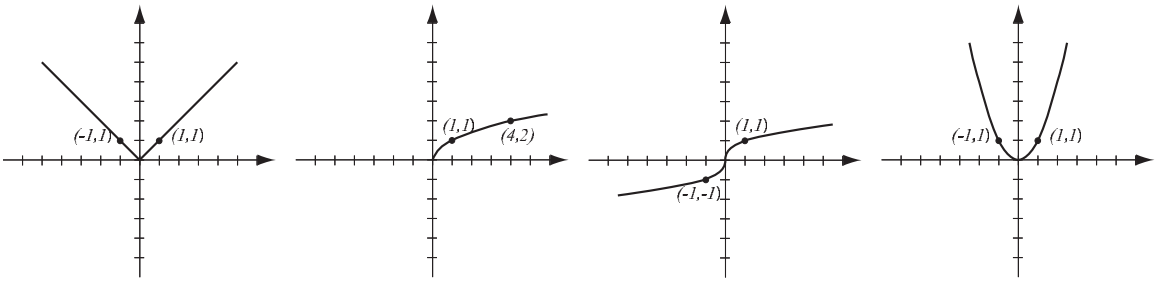
Bonus (10pts) Let $A = (1, 5)$ be a point in the plane. Find a point B on the x-axis so that the line through A and B is parallel to the line $y = 3x - 1$.

College Algebra — Exam 2
MAT 140, Fall 2020 — D. Ivanšić

Name: _____

Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (20pts) Let $f(x) = \dfrac{1}{x-4}$, $g(x) = \sqrt{3x-5}$.

Find the following (simplify where possible):

$(f+g)(3) =$ $(fg)(5) =$

$\dfrac{f}{g}(x) =$ $(f \circ g)(18) =$

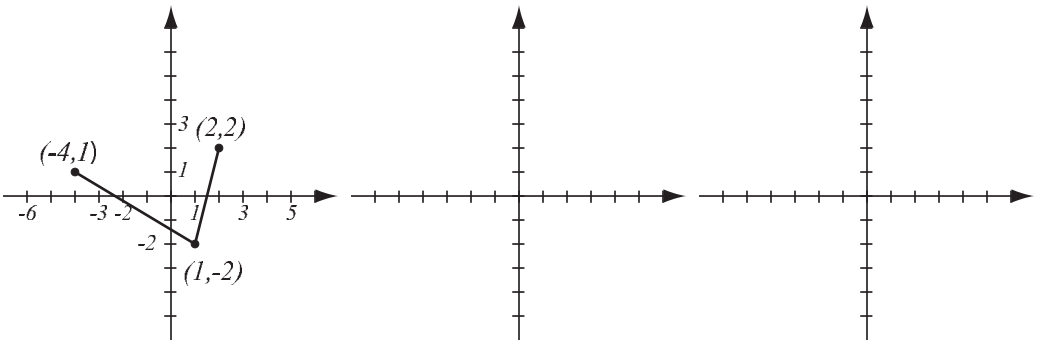
$(g \circ f)(x) =$

The domain of $\dfrac{g}{f}$ in interval notation

3. (6pts) Consider the function $h(x) = \dfrac{13}{(x+4)^2}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

4. (6pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = \sqrt{x}$, shifted 2 units to the left.
b) shape of $y = |x|$, stretched vertically by factor 3, then reflected over the x-axis.

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $2f(x+1)$ and $-f\left(\frac{1}{2}x\right)$ and label all the relevant points.



6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x - 1, & \text{if } -2 < x \leq 2 \\ 1 - x, & \text{if } x > 2 \end{cases}$$

7. (8pts) Find the values of the piecewise-defined function.

$$f(x) = \begin{cases} 3x - 2, & \text{if } x \leq -3 \\ x^2, & \text{if } -3 < x < 1 \\ |\sqrt{x} - 10|, & \text{if } 1 \leq x < 7 \end{cases}$$

$f(-20) =$ $f(9) =$

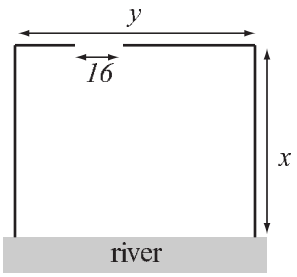
$f(4) =$ $f(-1) =$

8. (20pts) Let $f(x) = 0.1x^4 - 2x^2 + 3$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate units on the axes.
b) Determine algebraically whether the function is odd, even, or neither.
c) Verify your conclusion from b) by stating symmetry.
d) Find the local maxima and minima for this function.
e) State the intervals where the function is increasing and where it is decreasing.

9. (14pts) Farmer Felix is constructing a rectangular enclosure with area 10,000 square feet in a field along a river. The side along the river does not need fencing, and the enclosure has one 16-foot opening. Felix's goal is to minimize construction cost, same as minimizing the total length of the fence.

a) Express the total fence length as a function of the length of one of the sides x. What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the enclosure that has the smallest total fence length and what is the smallest total fence length?



Bonus. (10pts) Recall that the distance between points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Use this to find the point on the line $y = 3 - \frac{1}{2}x$ that is closest to the origin. *Hint: minimize the distance from a point (x, y) on the line to the origin. Make it a function only of x.*

College Algebra — Exam 3
MAT 140, Fall 2020 — D. Ivanšić

Name: _____

Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (5pts) $(2 + i)^3 =$

2. (5pts) $\dfrac{1 - 7i}{4 + 3i} =$

3. (4pts) Simplify and justify your answer.

$i^{270} =$

4. (6pts) Solve the equation by completing the square.

$x^2 + 10x + 13 = 0$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|x - 7| \geq 4$

6. (6pts) Let $P(x)$ be a polynomial of degree 3.
a) Draw a graph of P that has exactly 1 x-intercept and no turning points.
b) Draw a graph of P that has exactly 2 x-intercepts and the maximal number of turning points.

7. (12pts) The quadratic function $f(x) = x^2 - 6x + 10$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

Solve the equations:

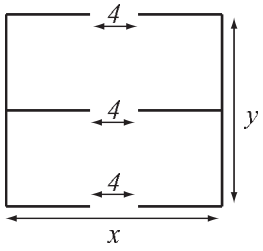
8. (8pts) $x^4 - 3x^2 - 28 = 0$

9. (8pts) $\sqrt{3x + 7} - x = 1$

10. (14pts) The polynomial $f(x) = (x+1)^2(x-4)$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

11. (12pts) Laura has a 25-meter-by-17-meter rectangular plot that she mows, and would like to decrease the area of the plot by shortening both sides of the rectangle by the same amount to get a rectangle with area that is 100 square meters smaller. By how much does Laura reduce each side of the rectangular plot?

12. (14pts) A hiking club is building a simple two-room hut as a refuge in the mountains with 4-foot openings left for doors. They have enough money to build 90 feet of walls, and their goal is to maximize the total area of the hut.
a) Express the total area of the hut as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the hut that has the biggest possible total area, and what is the biggest possible total area?



Bonus. (10pts) Find the quadratic function whose graph has y-intercept -2 and has $(1, 3)$ as the vertex .

College Algebra — Exam 4
MAT 140, Fall 2020 — D. Ivanšić

Name: _____

Show all your work!

1. (8pts) Evaluate without using the calculator:

$\log_4 64 =$ $\log_3 \frac{1}{27} =$ $\log_a \sqrt[5]{a^9} =$ $\log_{b^8} b^2 =$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_7 66$ with accuracy 6 decimal places. Show how you obtained your number.

3. (5pts) If $\log_a 3 = u$ and $\log_a 7 = v$, express in terms of u and v:

$\log_a \frac{3}{7} =$ $\log_a 63 =$

4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_4 \frac{16x^3}{\sqrt[4]{y^9}} =$

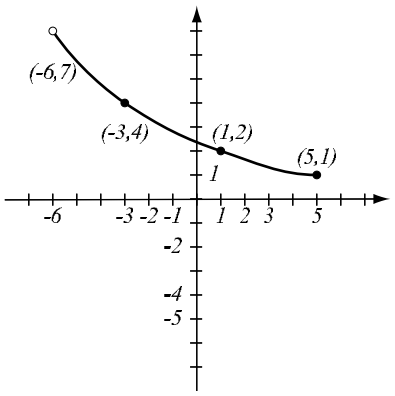
5. (6pts) Write as a single logarithm. Simplify if possible.

$4\ln(x^4y^{-3}) - 3\ln(x^4y^6) =$

6. (4pts) Simplify.

$\log 10^{3x-4} =$ $5^{\log_5 13} =$

7. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points, and showing any asymptotes.



8. (9pts) Let $f(x) = \dfrac{2x-1}{x}$.
a) Find the formula for f^{-1}.
b) Find the range of f.

9. (6pts) Using transformations, draw the graph of $f(x) = 3 - e^x$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.

10. (3pts) Find the domain of the function $f(x) = \log(-3x + 2)$ and write it in interval notation.

11. (9pts) \$2500 is deposited in an account bearing 2.34% interest, compounded monthly. How much is in the account after 10 years?

Solve the equations.

12. (6pts) $2^{5x+2} = \left(\frac{1}{8}\right)^{x-1}$

13. (8pts) $3^{2x-3} = 5^{x+4}$

14. (8pts) $\log_2(x + 2) + \log_2(x - 4) = 4$

15. (12pts) The population of Maricopa county, Arizona, was 3,072,000 in 2000 and 3,187,000 in 2010. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2000. Graph it on paper.
b) Find the predicted population in the year 2020.

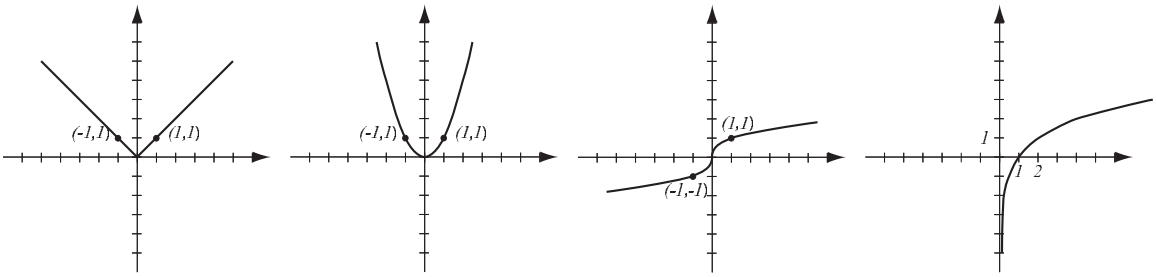
Bonus (10pts) Let $f(x) = x^2 - 6x + 15$ for $x \leq 3$.
a) Find the formula for f^{-1}. Completing the square may help, but it can be done in another way, too.
b) Find the range of f.

College Algebra — Final Exam
MAT 140, Fall 2020 — D. Ivanšić

Name: _____

Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



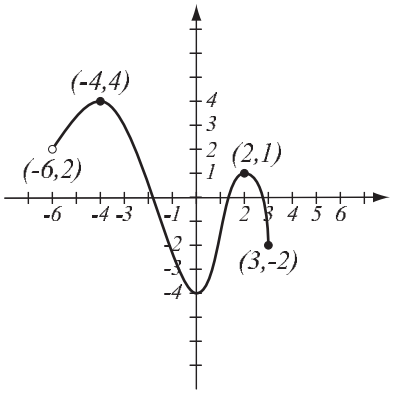
2. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find: $f(2) =$ $\qquad$ $f(-6) =$

b) What is the domain of f?

c) What is the range of f?

d) What are the solutions of the equation $f(x) = 3$?



3. (12pts) a) Write the equation of the line whose y-intercept is 2 and has slope 3.
b) Write the equation of the line through points $(-1, 3)$ and $(2, 2)$.
c) Are the two lines perpendicular?
d) Draw both lines.

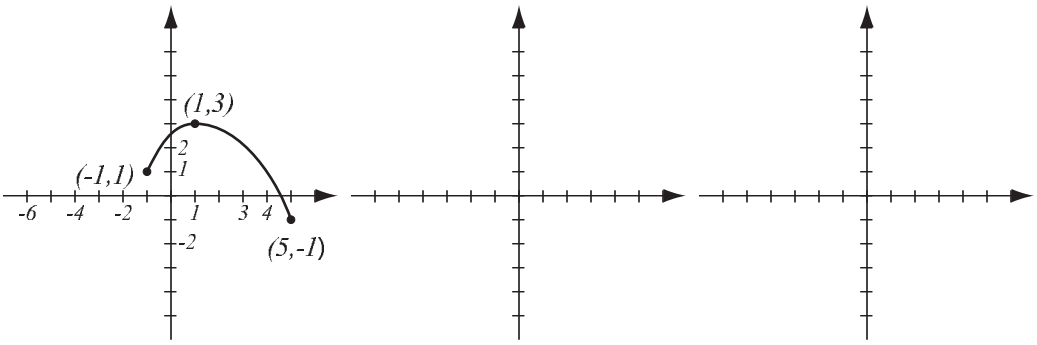
4. (6pts) Solve and write the solution in interval notation.

$|x - 7| < 4$

5. (4pts) Find the domain of the function $f(x) = \dfrac{1}{x^2 + 4x - 21}$ and write it in interval notation.

6. (6pts) Let $f(x) = \dfrac{4x}{x-3}$. Find the formula for f^{-1}.

7. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $2f(x+3)$ and $f(-x)+2$ and label all the relevant points.



8. (12pts) The quadratic function $f(x) = -x^2 - 4x + 5$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

9. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_2 \frac{32\sqrt[5]{y^8}}{x^9} =$$

10. (5pts) Write as a single logarithm. Simplify if possible.

$$3\log(x^{-2}y^4) - \log(x^3y^{-5}) =$$

11. (20pts) The polynomial $P(x) = x^3 - 25x$ is given (answer with 6 decimals accuracy).
a) What is the end behavior of the polynomial?
b) Factor the polynomial to find all the zeros and their multiplicities. Find the y-intercept.
c) Determine algebraically whether the function is odd, even, or neither.
d) Use the graphing calculator along with a) and b) to sketch the graph of P (yes, on paper!).
e) Verify your conclusion from c) by stating symmetry.
f) Find all the turning points (i.e., local maxima and minima).

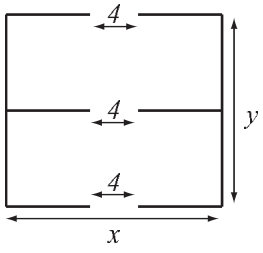
Solve the equations.

12. (8pts) $\dfrac{2x}{x+4} + \dfrac{10x-8}{x^2+2x-8} = \dfrac{x}{x-2}$

13. (6pts) $3^{x-1} = 9^{x-2}$

14. (14pts) Amy and Mitch bicycle along the same road. It takes Mitch 1 hour to travel the road. Amy leaves 15 minutes after Mitch, but gets to the end of the road at the same time as Mitch because she travels 3 mph faster than him.
a) What are the speeds of the cyclists?
b) How long is the road?

15. (14pts) A hiking club is building a simple two-room hut as a refuge in the mountains with 4-foot openings left for doors. They have enough money to build 120 feet of walls, and their goal is to maximize the total area of the hut.
a) Express the total area of the hut as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the hut that has the biggest possible total area, and what is the biggest possible total area?



16. (12pts) The population of Breedington was 11,000 in 2014 and 13,000 in 2018. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2014. Graph it on paper.
b) Find the predicted population in the year 2025.

Bonus (10pts) Let $A = (1, 5)$ be a point in the plane. Find a point B on the x-axis so that the line through A and B is parallel to the line $y = 3x - 1$.