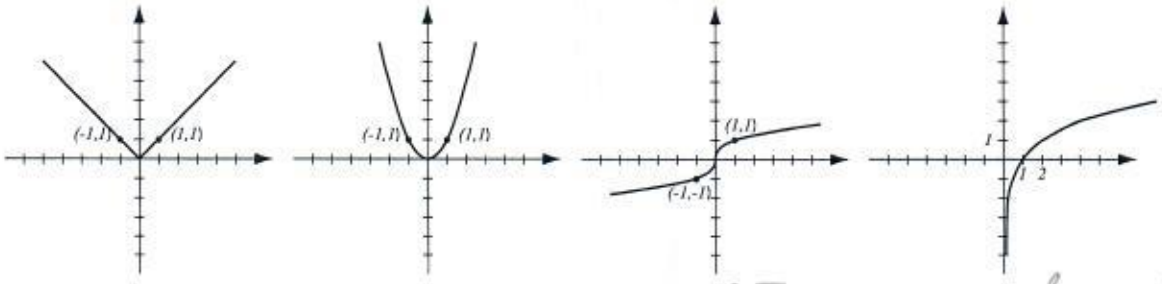
College Algebra — Final Exam
MAT 140, Fall 2020 — D. Ivanšić

Name: Saul Ocean

Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y = |x|$ $y = x^2$ $y = \sqrt[3]{x}$ $y = \log_a x,\ a > 1$

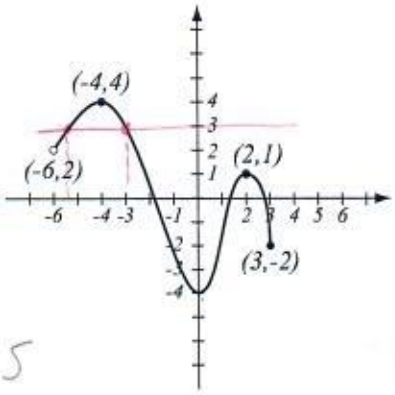
2. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find: $f(2) =$ 1 $f(-6) =$ not defined

b) What is the domain of f? $(-6, 3]$

c) What is the range of f? $[-4, 4]$

d) What are the solutions of the equation $f(x) = 3$? $x = -3, -5.5$



3. (12pts) a) Write the equation of the line whose y-intercept is 2 and has slope 3.
b) Write the equation of the line through points $(-1, 3)$ and $(2, 2)$.
c) Are the two lines perpendicular?
d) Draw both lines.

a) $y = 3x + 2$

b) $m = \frac{2-3}{2-(-1)} = -\frac{1}{3}$

$y - 3 = -\frac{1}{3}(x - (-1))$

$y - 3 = -\frac{1}{3}x - \frac{1}{3}$

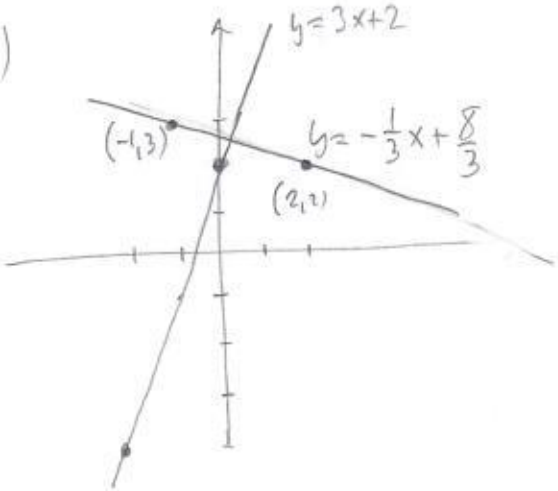
$y = -\frac{1}{3}x + \frac{8}{3}$

c) They are.

$m_1 \cdot m_2 = 3 \cdot (-\frac{1}{3})$

$= -1$

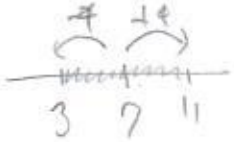
d)



4. (6pts) Solve and write the solution in interval notation.

$|x - 7| < 4$

distance from x to 7 < 4



$(3, 11)$

5. (4pts) Find the domain of the function $f(x) = \dfrac{1}{x^2 + 4x - 21}$ and write it in interval notation.

Can't have: $x^2+4x-21=0$

$(x+7)(x-3)=0$

$x=-7, 3$

$(-\infty,-7)\cup(-7,3)\cup(3,\infty)$

-7 $\quad$ 3

6. (6pts) Let $f(x) = \dfrac{4x}{x-3}$. Find the formula for f^{-1}.

$y = \dfrac{4x}{x-3}$

$(x-3)y = 4x$

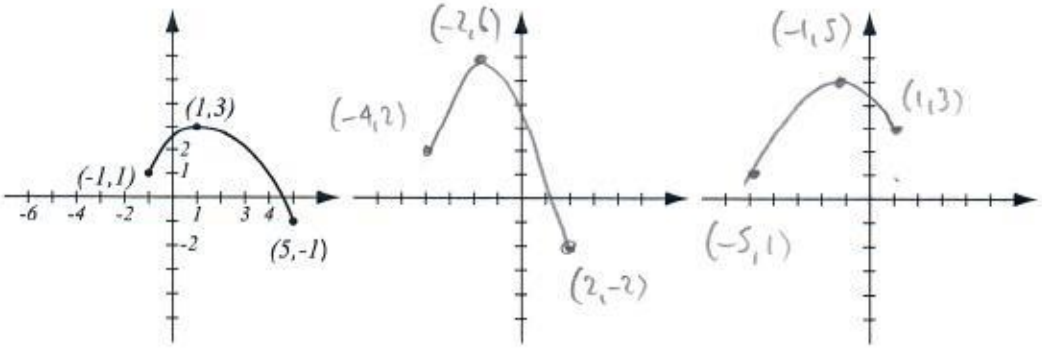
$xy - 3y = 4x$

$xy - 4x = 3y$

$x(y-4) = 3y$

$x = \dfrac{3y}{y-4} = f^{-1}(y)$

7. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $2f(x+3)$ and $f(-x)+2$ and label all the relevant points.



$y = 2f(x+3)$

shift left 3

stretch vert., factor = 2

$y = f(-x)+2$

reflect in y-axis

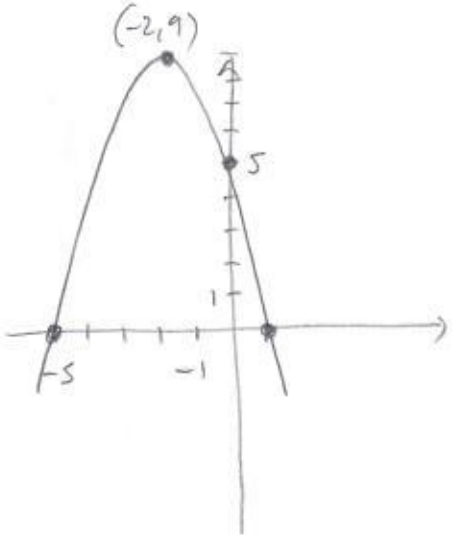
shift up 2

8. (12pts) The quadratic function $f(x) = -x^2 - 4x + 5$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.



a) $-x^2 + 4x + 5 = 0$

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x = -5, 1$

y-int. 5

b) $h = -\frac{b}{2a} = -\frac{-4}{2\cdot(-1)} = -2$

$k = f(-2) = -4 + 8 + 5 = 9$

9. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_2 \frac{32\sqrt[5]{y^8}}{x^9} = \log_2 32 + \log_2 y^{\frac{8}{5}} - \log_2 x^9$$

$$= 5 + \frac{8}{5}\log_2 y - 9\log_2 x$$

10. (5pts) Write as a single logarithm. Simplify if possible.

$$3\log(x^{-2}y^4) - \log(x^3y^{-5}) = \log(x^{-2}y^4)^3 - \log(x^3y^{-5})$$

$$= \log\frac{(x^{-2}y^4)^3}{x^3y^{-5}} = \log\frac{x^{-6}y^{12}}{x^3y^{-5}} = \log(x^{-9}y^{17}) = \log\frac{y^{17}}{x^9}$$

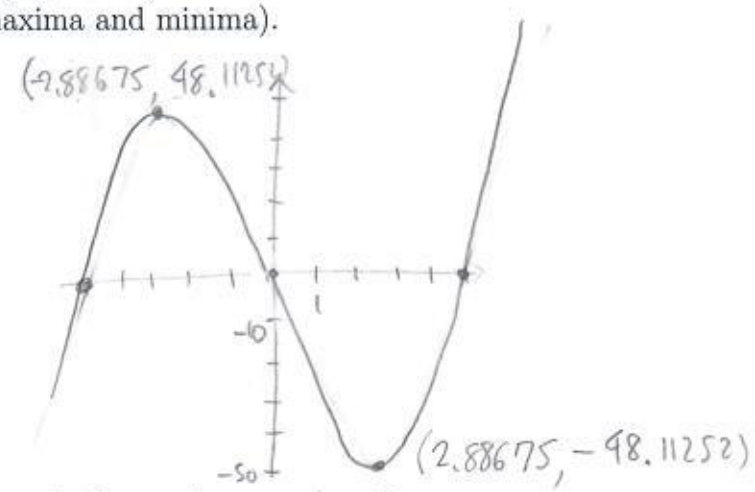
11. (20pts) The polynomial $P(x) = x^3 - 25x$ is given (answer with 6 decimals accuracy).
a) What is the end behavior of the polynomial?
b) Factor the polynomial to find all the zeros and their multiplicities. Find the y-intercept.
c) Determine algebraically whether the function is odd, even, or neither.
d) Use the graphing calculator along with a) and b) to sketch the graph of P (yes, on paper!).
e) Verify your conclusion from c) by stating symmetry.
f) Find all the turning points (i.e., local maxima and minima).

a) like x^3 ↗↘

b) $P(x) = x(x^2-25)$ y-int: 0

$= x(x-5)(x+5)$

zero	0	5	−5
mult.	1	1	1



c) $P(-x) = (-x)^3 - 25(-x)$

$= -x^3 + 25x$

$= -P(x)$ odd

e) Symmetric wrt origin

d) Local max is $P(-2.88675) = 48.11252$

Local min is $P(2.88675) = -48.11252$

Solve the equations.

12. (8pts) $\dfrac{2x}{x+4} + \dfrac{10x-8}{x^2+2x-8} = \dfrac{x}{x-2}$ $\quad | \cdot (x+4)(x-2)$

$(x+4)(x-2)$

$\frac{2x}{x+4}(x+4)(x-2) + \frac{10x-8}{(x+4)(x-2)}(x+4)(x-2) = \frac{x}{x-2}(x+4)(x-2)$

$2x(x-2) + (10x-8) = x(x+4)$

$2x^2 - 4x + 10x - 8 = x^2 + 4x \quad | -x^2 - 4x$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4, 2$ ← both give 0 in denom.

so no solutions

13. (6pts) $3^{x-1} = 9^{x-2}$

$3^{x-1} = (3^2)^{x-2}$

$3^{x-1} = 3^{2x-4}$

$x - 1 = 2x - 4$

$3 = x$

14. (14pts) Amy and Mitch bicycle along the same road. It takes Mitch 1 hour to travel the road. Amy leaves 15 minutes after Mitch, but gets to the end of the road at the same time as Mitch because she travels 3 mph faster than him.

a) What are the speeds of the cyclists?

b) How long is the road?

dist. rate, time

Amy: $d, r+3, 1-\frac{1}{4}$

Mitch $d, r, 1$

$d = (r+3)\frac{3}{4}$

$d = r \cdot 1$

$r \cdot 1 = (r+3)\frac{3}{4} \quad | \cdot 4$

$4r = 3r + 9$

$r = 9$ mph

$d = 9 \cdot 1 = 9$ miles

a) Amy rides 12 mph

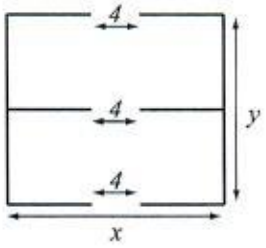
Mitch rides 9 mph

b) Road is 9 miles long.

15. (14pts) A hiking club is building a simple two-room hut as a refuge in the mountains with 4-foot openings left for doors. They have enough money to build 120 feet of walls, and their goal is to maximize the total area of the hut.

a) Express the total area of the hut as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the hut that has the biggest possible total area, and what is the biggest possible total area?



a) $A = xy = x\left(66 - \frac{3}{2}x\right) = -\frac{3}{2}x^2 + 66x$

$3(x-4) + 2y = 120$

$3x - 12 + 2y = 120$

$2y = 132 - 3x$

$y = 66 - \frac{3}{2}x$

Must have:

$x \geq 4$

$y \geq 0$

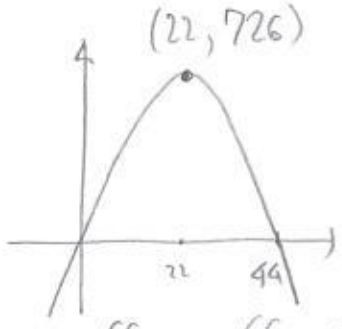
$66 - \frac{3}{2}x \geq 0$

$66 \geq \frac{3}{2}x \quad | \cdot \frac{2}{3}$

$44 \geq x$

Domain $[4, 44]$

b)



$h = -\frac{b}{2a} = -\frac{66}{2 \cdot (-\frac{3}{2})} = \frac{66}{3} = 22$

$k = 22 \cdot \left(66 - \frac{3}{2} \cdot 22\right) = 22 \cdot 33 = 726$

$66 - \frac{3}{2} \cdot 22$

Dimensions: 22×33 ft

Max area: 726 sq. ft

16. (12pts) The population of Breedington was 11,000 in 2014 and 13,000 in 2018. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2014. Graph it on paper.
b) Find the predicted population in the year 2025.

a) $P(t) = 11e^{kt}$

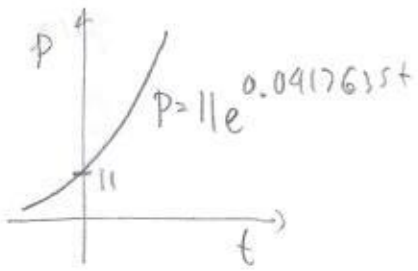
$13 = P(4) = 11e^{k\cdot 4}$

$\frac{13}{11} = e^{k\cdot 4}$ / ln

$\ln\frac{13}{11} = 4k$

$k = \frac{\ln\frac{13}{11}}{4} = 0.0417635$

$P(t) = 11\cdot e^{0.0417635t}$



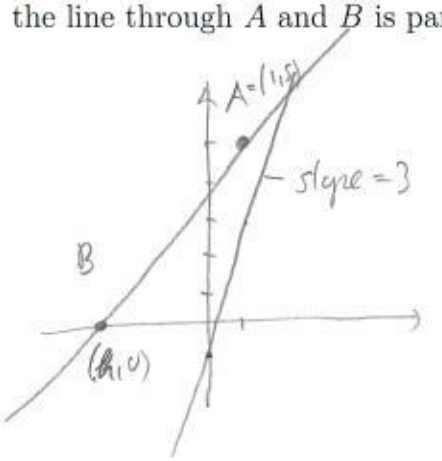
b) 2025 is 11 years after 2014

$P(11) = 11e^{0.0417635\cdot 11}$

$= 17.41434$

About 17,414 people in Breedington in 2025.

Bonus (10pts) Let $A = (1, 5)$ be a point in the plane. Find a point B on the x-axis so that the line through A and B is parallel to the line $y = 3x - 1$.



slope of AB is

$\frac{5-0}{1-h} = 3$

$\frac{5}{1-h} = 3$

$\frac{5}{3} = 1-h$

$h = 1 - \frac{5}{3}$

$h = -\frac{2}{3}$

$B = (-\frac{2}{3}, 0)$