College Algebra — Exam 4
MAT 140, Fall 2020 — D. Ivanšić

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Show all your work!

1. (8pts) Evaluate without using the calculator:

$\log_4 64 = 3$ $\quad 4^? = 64$

$\log_3 \frac{1}{27} = -3$ $\quad 3^? = \frac{1}{27} = \frac{1}{3^3} = 3^{-3}$

$\log_a \sqrt[5]{a^9} = \frac{9}{5}$ $\quad a^? = \sqrt[5]{a^9} = a^{\frac{9}{5}}$

$\log_{b^8} b^2 = \frac{1}{4}$ $\quad (b^8)^? = b^2$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_7 66$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_7 66 = \frac{\ln 66}{\ln 7} = 2.153057$

3. (5pts) If $\log_a 3 = u$ and $\log_a 7 = v$, express in terms of u and v:

$\log_a \frac{3}{7} = \log_a 3 - \log_a 7 = u - v$

$\log_a 63 = \log_a 7 + \log_a 3^2$
$7 \cdot 3^2$
$= \log_a 7 + 2\log_a 3$
$= v + 2u$

4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_4 \frac{16x^3}{\sqrt[4]{y^9}} = \log_4 16x^3 - \log_4 y^{\frac{9}{4}} = \log_4 16 + \log_4 x^3 - \log_4 y^{\frac{9}{4}}$
$= 2 + 3\log_4 x - \frac{9}{4}\log_4 y$

5. (6pts) Write as a single logarithm. Simplify if possible.

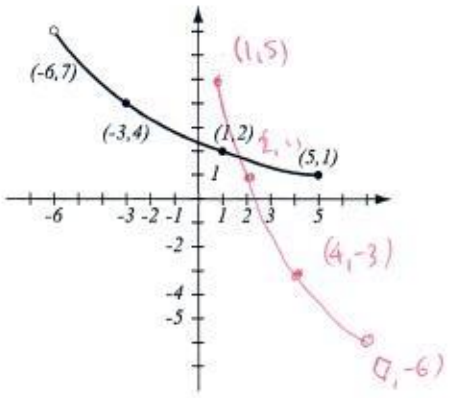
$4\ln(x^4y^{-3}) - 3\ln(x^4y^6) = \ln(x^4y^{-3})^4 - \ln(x^4y^6)^3 = \ln\frac{(x^4y^{-3})^4}{(x^4y^6)^3}$
$= \ln\frac{x^{16}y^{-12}}{x^{12}y^{18}} = \ln(x^4y^{-30}) = \ln\frac{x^4}{y^{30}}$

6. (4pts) Simplify.

$\log 10^{3x-4} = 3x-4$

$5^{\log_5 13} = 13$

7. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points, and showing any asymptotes.



a) Yes - it passes the horizontal line test

8. (9pts) Let $f(x) = \dfrac{2x-1}{x}$.
a) Find the formula for f^{-1}.
b) Find the range of f.

$y = \dfrac{2x-1}{x}$

$yx = 2x-1$

$yx - 2x = -1$

$(y-2)x = -1$

$x = -\dfrac{1}{y-2} = \dfrac{1}{2-y}$

range f = domain f^{-1} = $(-\infty, 2) \cup (2, \infty)$

↑

can't have $2-y=0$

$y=2$

9. (6pts) Using transformations, draw the graph of $f(x) = 3 - e^x$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.

$y = e^x$ reflect in x-axis $y = -e^x$ shift up 3 $y = -e^x + 3 = 3 - e^x$

10. (3pts) Find the domain of the function $f(x) = \log(-3x + 2)$ and write it in interval notation.

Must have: $-3x+2>0$ $\quad x < \frac{2}{3}$ $\quad \frac{2}{3}$

$-3x > -2$

$x < \frac{-2}{-3}$ $\quad (-\infty, \frac{2}{3})$

11. (9pts) \$2500 is deposited in an account bearing 2.34% interest, compounded monthly. How much is in the account after 10 years?

$$A = 2500\left(1 + \frac{0.0234}{12}\right)^{12 \cdot 10} = 3158.39$$

Solve the equations.

12. (6pts) $2^{5x+2} = \left(\frac{1}{8}\right)^{x-1}$

$2^{5x+2} = (2^{-3})^{x-1}$

$2^{5x+2} = 2^{-3x+3}$

$5x+2 = -3x+3$

$8x = 1$

$x = \frac{1}{8}$

13. (8pts) $3^{2x-3} = 5^{x+4}$ $\quad | \ln$

$\ln 3^{2x-3} = \ln 5^{x+4}$

$(2x-3)\ln 3 = (x+4)\ln 5$

$2\ln 3 \cdot x - 3\ln 3 = \ln 5 \cdot x + 4\ln 5$

$2\ln 3 \, x - \ln 5 \cdot x = 4\ln 5 + 3\ln 3$

$(2\ln 3 - \ln 5)x = 4\ln 5 + 3\ln 3$

$x = \frac{4\ln 5 + 3\ln 3}{2\ln 3 - \ln 5}$

$= 16.55973$

14. (8pts) $\log_2(x+2) + \log_2(x-4) = 4$

$\log_2((x+2)(x-4)) = 4$ $\quad | 2^{\square}$

$2^{\log_2((x+2)(x-4))} = 2^4$

$(x+2)(x-4) = 16$

$x^2 - 2x - 8 = 16$

$x^2 - 2x - 24 = 0$

$(x-6)(x+4) = 0$

$x = 6, \cancel{-4}$

gives neg. number in log

15. (12pts) The population of Maricopa county, Arizona, was 3,072,000 in 2000 and 3,187,000 in 2010. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2000. Graph it on paper.
b) Find the predicted population in the year 2020.

$P(t) = Pe^{kt} = 3072\, e^{kt}$

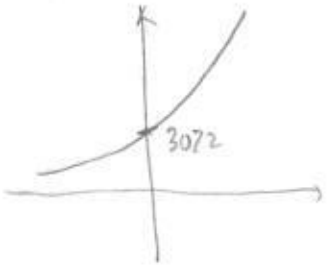
In thousands

$3187 = P(10) = 3072\, e^{k\cdot 10}$

$\frac{3187}{3072} = e^{k\cdot 10}$ | ln

$\ln \frac{3187}{3072} = k\cdot 10$

$k = \frac{\ln \frac{3187}{3072}}{10} = 0.00367512$



$P(20) = 3072\, e^{\cdot 20} = 3306.305013$

About 3,306,305 people in 2020.

Bonus (10pts) Let $f(x) = x^2 - 6x + 15$ for $x \leq 3$.
a) Find the formula for f^{-1}. Completing the square may help, but it can be done in another way, too.
b) Find the range of f.

$y = x^2 - 6x + 15$

$y + 3^2 = x^2 - 2\cdot x\cdot 3 + 3^2 + 15$

$y + 9 = (x-3)^2 + 15$

$y - 6 = (x-3)^2$

$\sqrt{y-6} = \pm(x-3)$

Since $x - 3 \leq 0$ and left side is positive, we take $-(x-3)$

$\sqrt{y-6} = -x + 3$

$x = 3 - \sqrt{y-6} = f^{-1}(y)$

Range of f = domain of $f^{-1} = [6, \infty)$

Must have $y - 6 \geq 0$

$y \geq 6$