College Algebra — Exam 3 MAT 140, Fall 2020 — D. Ivanšić	**Name:** Saul Ocean *Show all your work!*

Simplify, so that the answer is in form $a + bi$.

1. (5pts) $(2+i)^3 =$ $(2+i)^2\cdot(2+i) = (4+4i+i^2)(2+i) = (3+4i)(2+i)$

$= 6+8i+3i+4i^2 = 2+11i$

2. (5pts) $\dfrac{1-7i}{4+3i} =$ $\dfrac{1-7i}{4+3i}\cdot\dfrac{4-3i}{4-3i} = \dfrac{4-28i-3i+21i^2}{4^2-(3i)^2} = \dfrac{-17-31i}{16+9} = -\dfrac{17}{25}-\dfrac{31}{25}i$

3. (4pts) Simplify and justify your answer.

$i^{270} =$ $i^{268}\cdot i^2 = (i^4)^{67}\cdot i^2 = 1\cdot(-1) = -1$

$270 \div 4 = 67$, rem 2

4. (6pts) Solve the equation by completing the square.

$x^2 + 10x + 13 = 0$ $\quad | +5^2$

$x^2+2\cdot x\cdot 5+5^2+13 = 5^2 \quad | -13$

$(x+5)^2 = 12$

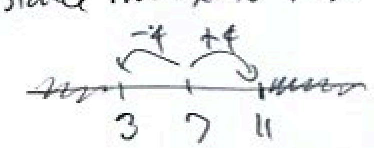
$x+5 = \pm\sqrt{12}$

$x = -5 \pm 2\sqrt{3}$

5. (6pts) Solve the inequality. Write the solution in interval form.

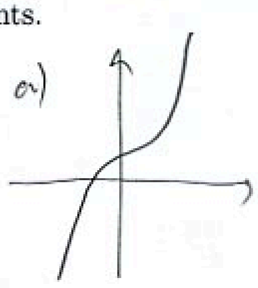
$|x-7| \geq 4$

distance from x to $7 \geq 4$ $\quad (-\infty, 3] \cup [11, \infty)$

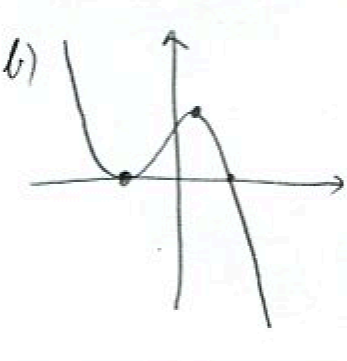


6. (6pts) Let $P(x)$ be a polynomial of degree 3.
a) Draw a graph of P that has exactly 1 x-intercept and no turning points.
b) Draw a graph of P that has exactly 2 x-intercepts and the maximal number of turning points.

a)



b)



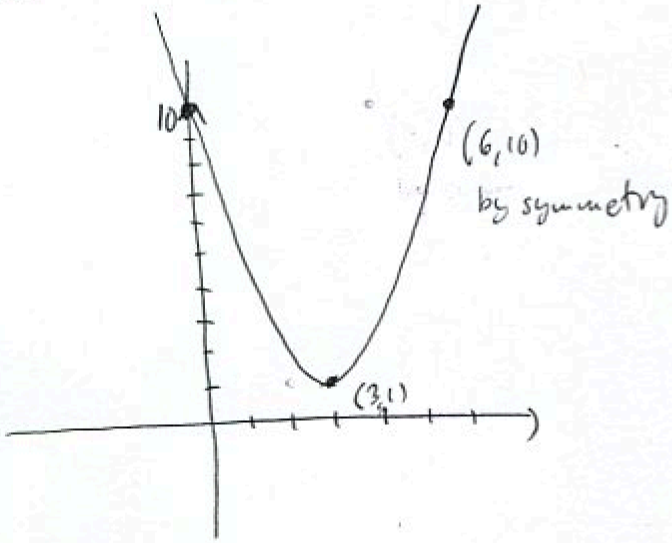
max no of turning pts = 2

7. (12pts) The quadratic function $f(x) = x^2 - 6x + 10$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.



a) $x^2 - 6x + 10 = 0$

$x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 10}}{2 \cdot 1}$

$= \dfrac{6 \pm \sqrt{-4}}{2}$ not real, so no x-int.

$f(0) = 10$,

b) $h = -\dfrac{b}{2a} = -\dfrac{-6}{2 \cdot 1} = 3$

$k = f(3) = 9 - 18 + 10 = 1$

Solve the equations:

8. (8pts) $x^4 - 3x^2 - 28 = 0$

$(x^2)^2 - 3x^2 - 28 = 0$

Let $u = x^2$ $\quad u^2 - 3u - 28 = 0$

$(u-7)(u+4) = 0$

$u = 7, -4$

$x^2 = 7$ or $x^2 = -4$

$x = \pm\sqrt{7}$ $\quad x = \pm 2i$

9. (8pts) $\sqrt{3x+7} - x = 1$

$\sqrt{3x+7} = x + 1 \quad |^2$

$3x + 7 = x^2 + 2 \cdot x \cdot 1 + 1^2 \quad | -3x$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3, -2$

Check: $x = 3$ $\quad \sqrt{9+7} - 3 \stackrel{?}{=} 1$

$\sqrt{16} - 3 \stackrel{?}{=} 1$ yes

$x = -2$ $\quad \sqrt{3(-2)+7} - (-2) \stackrel{?}{=} 1$

$\sqrt{1} + 2 \stackrel{?}{=} 1$ no

Sol: $\boxed{x = 3}$

10. (14pts) The polynomial $f(x) = (x+1)^2(x-4)$ is given.

a) What is the end behavior of the polynomial?

b) List all the zeros and their multiplicities. Find the y-intercept.

c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).

d) Find all the turning points (i.e., local maxima and minima).

a) $f(x) = (x+\ldots)^2(x-\ldots) = x^2 \cdot x = x^3 + \ldots$

Behaves like x^3

b)

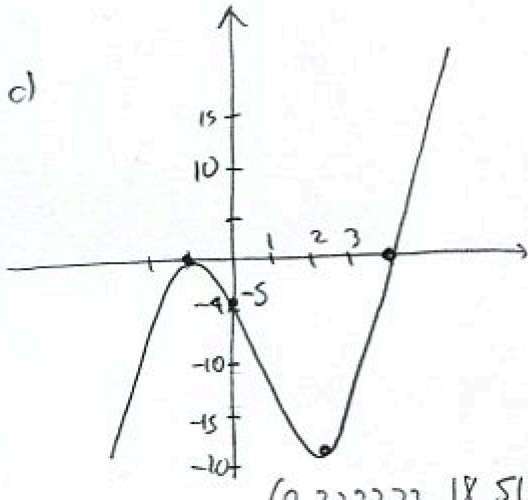
zeros	-1	4
mult.	2	1

y-int: $f(0) = 1^2 \cdot (-4) = -4$

d) Turning Points:

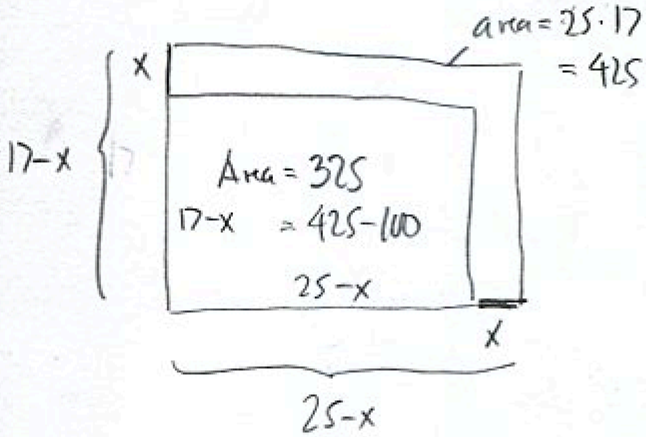
$(-1, 0)$, $(2.333333, 18.51852)$

c)



$(2.333333, 18.51852)$

11. (12pts) Laura has a 25-meter-by-17-meter rectangular plot that she mows, and would like to decrease the area of the plot by shortening both sides of the rectangle by the same amount to get a rectangle with area that is 100 square meters smaller. By how much does Laura reduce each side of the rectangular plot?



$(25-x)(17-x) = 325$

$425 - 17x - 25x + x^2 = 325$

$x^2 - 42x + 100 = 0$

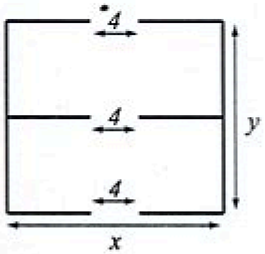
$x = \frac{-(-42) \pm \sqrt{(-42)^2 - 4 \cdot 1 \cdot 100}}{2 \cdot 1} = \frac{42 \pm \sqrt{1364}}{2}$ 4·341

$= \frac{42 \pm 2\sqrt{341}}{2} = 21 \pm \sqrt{341} = 39.466185, 2.533815$

cannot reduce by more than 17

She reduces the plot by 2.533815 meters on each side

12. (14pts) A hiking club is building a simple two-room hut as a refuge in the mountains with 4-foot openings left for doors. They have enough money to build 90 feet of walls, and their goal is to maximize the total area of the hut.
a) Express the total area of the hut as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the hut that has the biggest possible total area, and what is the biggest possible total area?



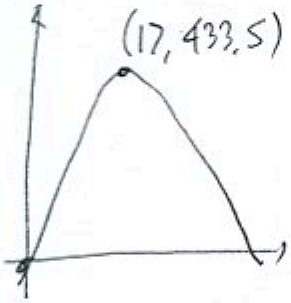
a) $A = xy = x\cdot\left(51 - \frac{3}{2}x\right)^2 = -\frac{3}{2}x^2 + 51x$

$3(x-4) + 2y = 90$

$3x - 12 + 2y = 90$

$2y = 102 - 3x$

$y = 51 - \frac{3}{2}x$



Domain:
Must have:

$x \geq 4$

$y \geq 0$

$51 - \frac{3}{2}x \geq 0$

$51 \geq \frac{3}{2}x \quad | \cdot \frac{2}{3}$

$34 \geq x$

$[4, 34]$

$h = -\frac{b}{2a} = -\frac{51}{2\cdot\left(-\frac{3}{2}\right)} = \frac{51}{3} = 17$

$k = -\frac{3}{2}\cdot 17^2 + 51\cdot 17 = 433.5$

Dimensions are 17×25.5 ft

Max area is 433.5 square ft.

Bonus. (10pts) Find the quadratic function whose graph has y-intercept -2 and has $(1,3)$ as the vertex .

$y = a(x-h)^2 + k$

$y = a(x-1)^2 + 3$

When $x=0$, $y=-2$

$-2 = a(0-1)^2 + 3$

$-2 = a + 3$

$a = -5$

$f(x) = -5(x-1)^2 + 3$

OR: $y = ax^2 + bx + c$

When $x=0$ $\quad -2 = 0 + 0 + c$

so $c = -2$

$1 = -\frac{b}{2a}$ so $b = -2a$

$x=1, y=3$ gives $\quad 3 = a\cdot 1 + b - 2$

$3 = a - 2a - 2$

$5 = -a$, $\quad b = 10$

$a = -5$

$y = -5x^2 + 10x - 2$