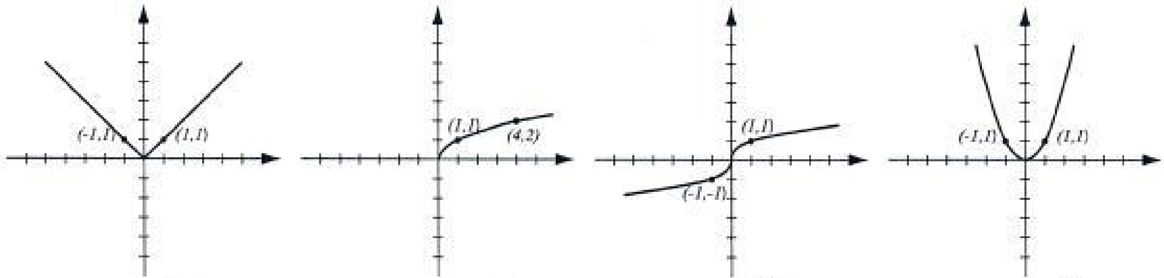
College Algebra — Exam 2 **MAT 140, Fall 2020 — D. Ivanšić**	**Name:** Saul Ocean *Show all your work!*

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y=|x|$ $\qquad$ $y=\sqrt{x}$ $\qquad$ $y=\sqrt[3]{x}$ $\qquad$ $y=x^2$

2. (20pts) Let $f(x)=\dfrac{1}{x-4}$, $g(x)=\sqrt{3x-5}$.

Find the following (simplify where possible):

$(f+g)(3) = f(3)+g(3)$

$= \dfrac{1}{3-4} + \underbrace{\sqrt{3\cdot 3-5}}_{4} = -1+2 = 1$

$(fg)(5) = f(5)\cdot g(5) = \dfrac{1}{5-4}\cdot\sqrt{3\cdot 5-5}$

$= 1\cdot\sqrt{10} = \sqrt{10}$

$\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{\frac{1}{x-4}}{\sqrt{3x-5}} = \dfrac{1}{x-4}\cdot\dfrac{1}{\sqrt{3x-5}}$

$= \dfrac{1}{(x-4)\sqrt{3x-5}}$

$(f\circ g)(18) = f(g(18)) = f\left(\sqrt{3\cdot 18-5}\right)$

$= f(\sqrt{49}) = f(7) = \dfrac{1}{7-4} = \dfrac{1}{3}$

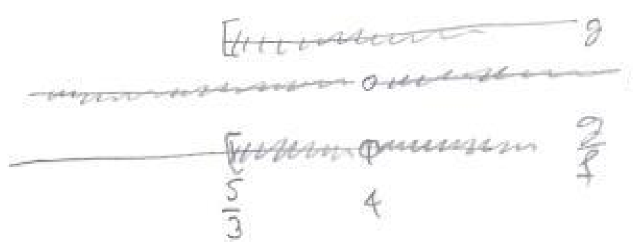
$(g\circ f)(x) = g(f(x)) = g\left(\dfrac{1}{x-4}\right)$

$= \sqrt{3\cdot\dfrac{1}{x-4}-5} = \sqrt{\dfrac{3}{x-4}-5}$

The domain of $\dfrac{g}{f}$ in interval notation

domain of g:
must have $3x-5\geq 0$
$3x\geq 5$
$x\geq\dfrac{5}{3}$

domain of f:
can't have
$x-4=0$
$x=4$
$x\neq 4$



Domain of $\dfrac{g}{f}$ is $\left[\dfrac{5}{3},4\right)\cup(4,\infty)$

Can't have $f(x)=0$ $\quad \dfrac{1}{x-4}=0$
never the case (no sol.)

3. (6pts) Consider the function $h(x) = \dfrac{13}{(x+4)^2}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

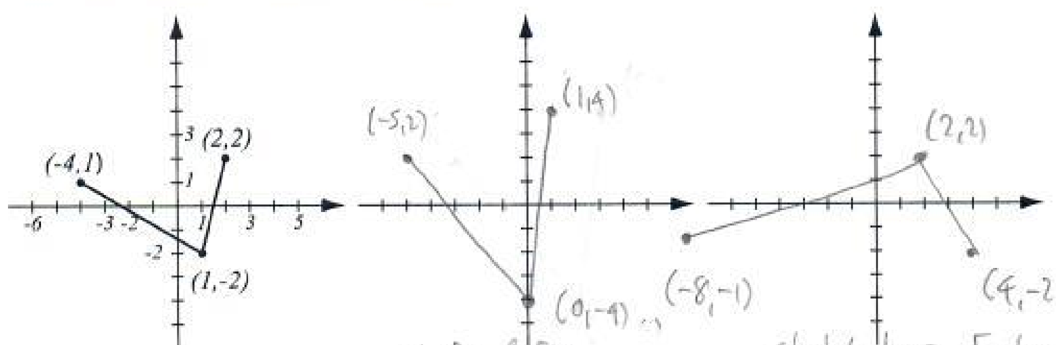
$g(x) = (x+4)^2$ $\qquad$ $g(x) = x+4$

$f(x) = \dfrac{13}{x}$ $\qquad$ $f(x) = \dfrac{13}{x^2}$

4. (6pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = \sqrt{x}$, shifted 2 units to the left.
b) shape of $y = |x|$, stretched vertically by factor 3, then reflected over the x-axis.

a) $y = \sqrt{x+2}$ $\qquad$ b) $y = -3|x|$

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $2f(x+1)$ and $-f\left(\frac{1}{2}x\right)$ and label all the relevant points.



shift left 1
stretch vert., factor = 2

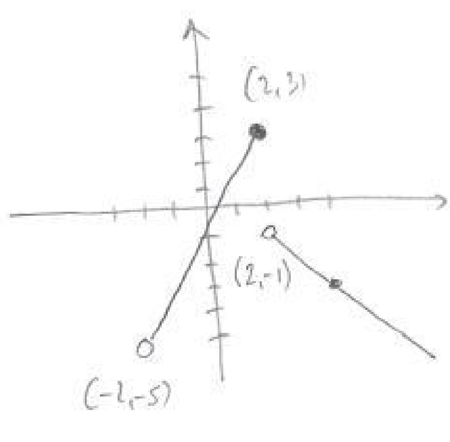
stretch horiz., factor = 2
reflect in x-axis

6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x-1, & \text{if } -2 < x \leq 2 \\ 1-x, & \text{if } x > 2 \end{cases}$$

x	2x-1
-2	-5
2	3

x	1-x
2	-1
4	-3



7. (8pts) Find the values of the piecewise-defined function.

$$f(x) = \begin{cases} 3x - 2, & \text{if } x \le -3 \\ x^2, & \text{if } -3 < x < 1 \\ |\sqrt{x} - 10|, & \text{if } 1 \le x < 7 \end{cases}$$

$f(-20) =$ $3(-20)-2 = -62$

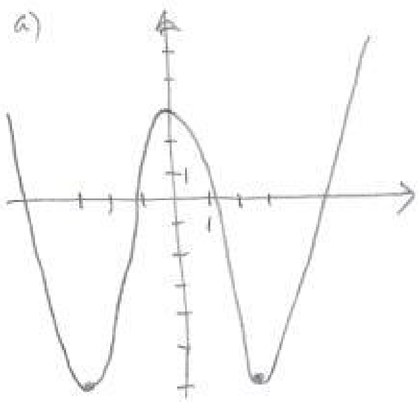
$f(9) =$ not defined

$f(4) =$ $|\sqrt{4}-10| = |-8| = 8$

$f(-1) =$ $(-1)^2 = 1$

8. (20pts) Let $f(x) = 0.1x^4 - 2x^2 + 3$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate units on the axes.
b) Determine algebraically whether the function is odd, even, or neither.
c) Verify your conclusion from b) by stating symmetry.
d) Find the local maxima and minima for this function.
e) State the intervals where the function is increasing and where it is decreasing.

a)



d) $f(-3.162277) = -7$ is a local min
$f(3.162277) = -7$ is a local min
$f(0) = 3$ is a local max.

e) decreasing on
$(-\infty, -3.162277)$, $(0, 3.162277)$
increasing on
$(-3.162277, 0)$ and $(3.162277, \infty)$

b) $f(-x) = 0.1(-x)^4 - 2(-x)^2 + 3$
$= 0.1x^4 - 2x^2 + 3$
$= f(x)$

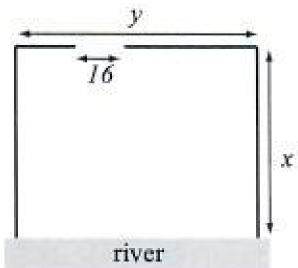
function is even

c) symmetric wrt y-axis

9. (14pts) Farmer Felix is constructing a rectangular enclosure with area 10,000 square feet in a field along a river. The side along the river does not need fencing, and the enclosure has one 16-foot opening. Felix's goal is to minimize construction cost, same as minimizing the total length of the fence.

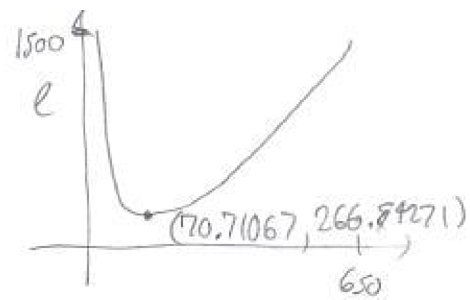
a) Express the total fence length as a function of the length of one of the sides x. What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the enclosure that has the smallest total fence length and what is the smallest total fence length?



$A = 10{,}000 = x \cdot y$ so $y = \frac{10000}{x}$

$\ell = 2x + y - 16 = 2x + \frac{10000}{x} - 16$



Domain,
Must have:

$x > 0$

$y \geq 16$

$\frac{10000}{x} \geq 16$

$10000 \geq 16x$

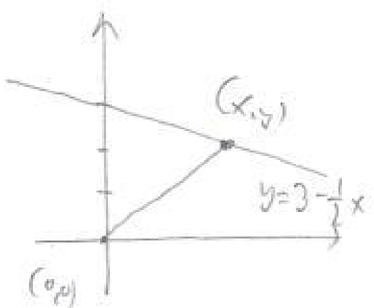
$x \leq \frac{10000}{16} = 625$

Domain: $(0, 625]$

dimensions $x \times y = 70.71067 \times 141.421373$ ft

Minimal Fence Length $= 266.84271$ ft

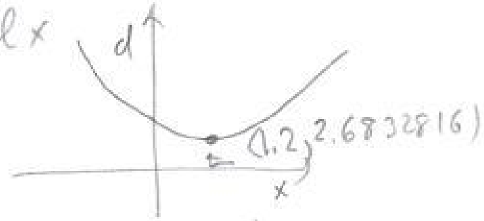
Bonus. (10pts) Recall that the distance between points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Use this to find the point on the line $y = 3 - \frac{1}{2}x$ that is closest to the origin. *Hint: minimize the distance from a point (x, y) on the line to the origin. Make it a function only of x.*



Distance from (x, y) to $(0,0)$ is $\sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$

Since pt. is on line $y = 3 - \frac{1}{2}x$, this is $d(x) = \sqrt{x^2 + (3 - \frac{1}{2}x)^2}$

Domain: all x



Closest point is $(1.2, 2.4)$

$3 - \frac{1}{2} \cdot 1.2$