

1. (8pts) Use the graph of the function f at right to answer the following questions.

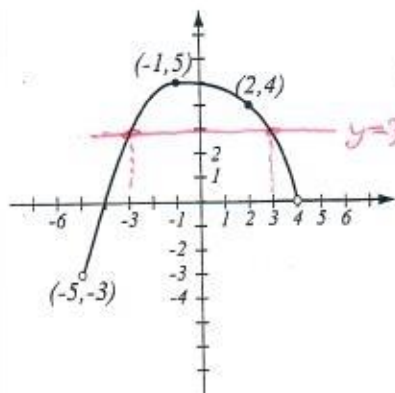
a) Find: $f(2) = 4$ $f(-6) = \text{not defined}$

b) What is the domain of f ? $(-5, 4)$

c) What is the range of f ? $(-3, 5)$

d) What are the solutions of the equation $f(x) = 3$?

$$x = -3, 3$$



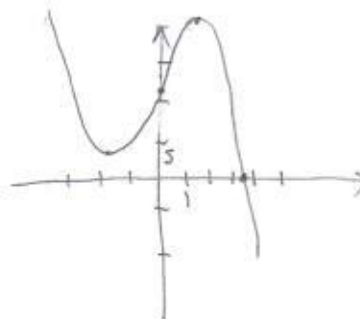
2. (10pts) Use your calculator to accurately sketch the graph of $y = -x^3 + 8x + 12$.

a) Draw the graph on paper and indicate units on the axes.

b) Find all the x - and y -intercepts (accuracy: 6 decimal points).

$$y\text{-int: } y = 12$$

$$x\text{-int: } 3.396096$$



3. (5pts) Write the equation of the line whose x -intercept is 3, and y -intercept is -5.

Goes through $(3, 0)$ slope: $\frac{-5 - 0}{0 - 3} = \frac{5}{3}$

$$(0, -5) \quad y = \frac{5}{3}x - 5$$

4. (10pts) Find the equation of the line (in form $y = mx + b$) that is perpendicular to the line $x + 4y = -8$ and passes through $(2, -1)$. Draw both lines.

$$x + 4y = -8$$

$$4y = -x - 8 \quad | \div 4$$

$$y = -\frac{x}{4} - 2$$

$$\text{slope} = -\frac{1}{4}$$

slope of perp. line

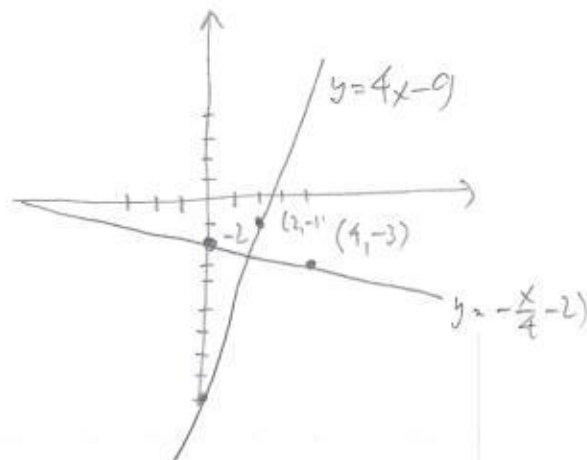
$$\text{is } -\frac{1}{-\frac{1}{4}} = 4$$

Perp. line:

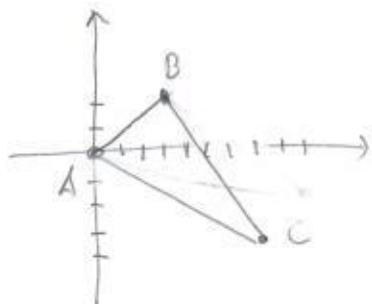
$$y - (-1) = 4(x - 2)$$

$$y = 4x - 8 - 1$$

$$y = 4x - 9$$



5. (8pts) Draw the triangle with vertices $A = (0,0)$, $B = (3,2)$ and $C = (7,-4)$ in the coordinate plane. Use the Pythagorean theorem to determine if the triangle is a right triangle.



$$d_{AB} = \sqrt{(3-0)^2 + (2-0)^2} = \sqrt{9+4} = \sqrt{13}$$

$$d_{BC} = \sqrt{(7-3)^2 + (-4-2)^2} = \sqrt{16+36} = \sqrt{52}$$

$$d_{AC} = \sqrt{(7-0)^2 + (-4-0)^2} = \sqrt{49+16} = \sqrt{65}$$

$$\sqrt{13}^2 + \sqrt{52}^2 = \sqrt{65}^2$$

$13 + 52 = 65$ yes, it is a right triangle

6. (9pts) Let $f(x) = x^2 + 4x - \frac{1}{\sqrt{x+1}}$. Find the following (simplify where appropriate).

$$f(3) = 3^2 + 4 \cdot 3 + \frac{1}{\sqrt{3+1}} = 9 + 12 + \frac{1}{2} = 21\frac{1}{2} = \frac{43}{2}$$

$$f(-6) = (-6)^2 + 4 \cdot (-6) - \frac{1}{\sqrt{-6+1}} \quad \text{not defined since } \sqrt{-5} \text{ is not defined}$$

$$f(\sqrt{u}) = \sqrt{u}^2 + 4\sqrt{u} - \frac{1}{\sqrt{\sqrt{u}+1}}$$

$$= u + 4\sqrt{u} - \frac{1}{\sqrt{\sqrt{u}+1}}$$

$$f(t-3) = (t-3)^2 + 4(t-3) - \frac{1}{\sqrt{t-3+1}}$$

$$= t^2 - 6t + 9 + 4t - 12 - \frac{1}{\sqrt{t-2}}$$

$$= t^2 - 2t - 3 - \frac{1}{\sqrt{t-2}}$$

7. (9pts) Find the domains of the functions below and write them using interval notation.

$$f(x) = \frac{x-3}{2x-5}$$

$$g(x) = \frac{\sqrt{5-3x}}{x+8}$$

Can't have $2x-5=0$

$$2x=5$$

$$x = \frac{5}{2}$$

~~cannot have~~
 $\frac{5}{2}$

$$(-\infty, \frac{5}{2}) \cup (\frac{5}{2}, \infty)$$

Must have:

$$5-3x \geq 0$$

$$5 \geq 3x \quad | \div 3$$

$$x \leq \frac{5}{3}$$

Can't have

$$x+8=0$$

$$x = -8$$

~~cannot have~~
 -8 $\frac{5}{3}$

$$(-\infty, -8) \cup (-8, \frac{5}{3}]$$

11. (14pts) Amy and Mitch bicycle along the same road. It takes Mitch 1 hour to travel the road. Amy leaves 12 minutes after Mitch, but gets to the end of the road at the same time as Mitch because she travels 2 mph faster than him.

a) What are the speeds of the cyclists?

b) How long is the road?

	distance	rate	time
Mitch	d	r	1
Amy	d	$r+2$	$1 - \frac{12}{60} = 1 - \frac{1}{5} = \frac{4}{5}$

a) Mitch: 8 mph

Amy: 10 mph

b) $d = 8 \cdot 1 = 8$ miles

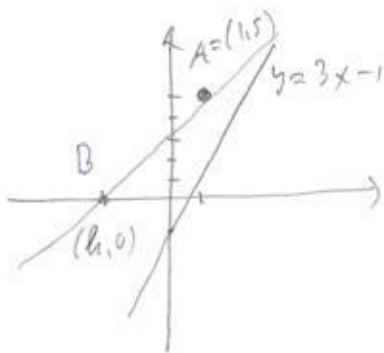
same $\left\{ \begin{array}{l} d = r \cdot 1 \\ d = (r+2) \cdot \frac{4}{5} \end{array} \right.$

$$r = (r+2) \cdot \frac{4}{5} \quad | \cdot 5$$

$$5r = 4r + 8 \quad | -r$$

$$r = 8 \text{ mph}$$

Bonus (10pts) Let $A = (1, 5)$ be a point in the plane. Find a point B on the x -axis so that the line through A and B is parallel to the line $y = 3x - 1$.



slope of $AB = 3$

$$\frac{0-5}{h-1} = 3$$

$$-5 = 3(h-1)$$

$$-5 = 3h - 3$$

$$3h = -2$$

$$h = -\frac{2}{3}$$

$$B = \left(-\frac{2}{3}, 0\right)$$

8. (5pts) Solve and write the solution in interval notation.

$$-2 \leq 5 - 2x < 4 \quad | -5 \quad \frac{7}{2} \geq x > \frac{1}{2} \quad \left(\frac{1}{2}, \frac{7}{2}\right]$$

$$-7 \leq -2x < -1 \quad | \div -2 \quad \left(\frac{1}{2}, \frac{7}{2}\right]$$

9. (10pts) The diameter of a circle has endpoints $(-3, 2)$ and $(1, 0)$.

a) Find the equation of the circle.

b) Draw the circle in the coordinate plane.

Center = midpoint of $(-3, 2), (1, 0)$

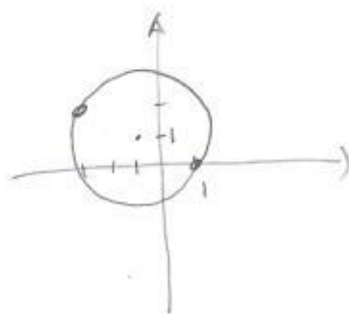
$$= \left(\frac{-3+1}{2}, \frac{2+0}{2}\right) = (-1, 1)$$

r = distance from center to $(1, 0)$:

$$= \sqrt{(1-(-1))^2 + (0-1)^2} = \sqrt{4+1} = \sqrt{5}$$

$$\text{Eq. of circle: } (x-(-1))^2 + (y-1)^2 = \sqrt{5}^2$$

$$(x+1)^2 + (y-1)^2 = 5$$



10. (12pts) Ellen plans to invest \$12,000: part at 3.5% simple interest, and the rest at 4.5% simple interest. What is the most she can invest at 3.5% to guarantee receiving \$500 in interest in a year? Solve as an inequality.

x = amount invested at 3.5%

$12,000 - x$ = amount invested at 4.5%

interest from 3.5% inv. + interest from 4.5% inv ≥ 500

$$0.035x \cdot 1 + 0.045 \cdot (12000 - x) \cdot 1 \geq 500$$

$$0.035x + 540 - 0.045x \geq 500 \quad | -500$$

$$-0.01x + 540 \geq 500 \quad | -500 + 0.01x$$

$$40 \geq 0.01x$$

$$x \leq \frac{40}{0.01} = 4000$$

Ellen can invest at most 4000 at 3.5%