Calculus 1 — Exam 1	**Name:** ____________________
MAT 250, Fall 2019 — D. Ivanšić	*Show all your work!*

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x \to -2+} f(x) =$

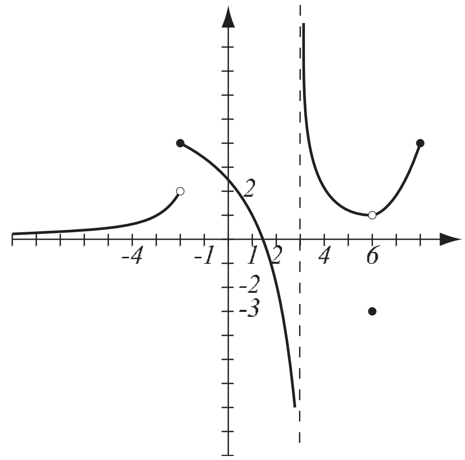
$\lim_{x \to -2^-} f(x) =$

$\lim_{x \to -2} f(x) =$

$\lim_{x \to -\infty} f(x) =$

$\lim_{x \to 6} f(x) =$

$\lim_{x \to 3} f(x) =$



List points in domain of f where f is not continuous and justify why it is not continuous at those points.

2. (6pts) Let $\lim_{x \to 5} f(x) = 3$ and $\lim_{x \to 5} g(x) = 1$. Use limit laws to find the limit below and show each step.

$$\lim_{x \to 5} \frac{xf(x) - 9g(x)}{f(x)^2 - 3} =$$

3. (10pts) Find $\lim_{x \to 0} x^4 \cdot (1 + \sin\left(\frac{1}{x}\right))^2$. Use the theorem that rhymes with insects that you might find on dogs and cats.

Find the following limits algebraically. Do not use the calculator.

4. (7pts) $\lim\limits_{x\to\infty} \dfrac{x^2-5x+7}{x+4} =$

5. (5pts) $\lim\limits_{x\to 7} \dfrac{x^2-6x-7}{x^2-11x+28} =$

6. (7pts) $\lim\limits_{x\to 5} \dfrac{x^2-25}{\sqrt{x}-\sqrt{5}} =$

7. (6pts) $\lim\limits_{x\to -3^-} \dfrac{x}{x+3} =$

8. (7pts) $\lim\limits_{x\to 1^+} \left(\dfrac{1}{x-1} - \dfrac{2}{x^2-1}\right) =$

9. (8pts) The equation $x^3 - x^2 + x = \sqrt{x} + 1$ is given. Use the Intermediate Value Theorem to show it has a solution in the interval $(0, 4)$.

10. (8pts) Explain in an intuitive way why the Intermediate Value Theorem is true on this example: Let f be a continuous function defined on the interval $[2, 5]$, and let $f(2) = -1$ and $f(5) = 4$. Justify graphically why there has to be a number c in the interval $(2, 5)$ so that $f(c) = 3$. (You need a picture and a nice sentence.)

11. (10pts) Consider the limit $\lim_{x \to 2} \dfrac{2^x - 4}{x - 2}$. Use your calculator (don't forget parentheses) to estimate this limit with accuracy 3 decimal points. Write a table of values that will support your answer.

x	$\dfrac{2^x - 4}{x - 2}$

x	$\dfrac{2^x - 4}{x - 2}$

12. (10pts) Is the function defined below continuous? Justify.

$$f(x) = \begin{cases} \dfrac{\sin(2x-6)}{x-3}, & \text{if } x \neq 3 \\ \\ 1, & \text{if } x = 3. \end{cases}$$

Bonus. (10pts) Show by example that the conclusion of the Intermediate Value Theorem is not true if the function is not continuous. Draw a function defined on the interval $[2, 5]$ for which $f(2) = -1$ and $f(5) = 4$. but there is no number c in the interval $(2, 5)$ so that $f(c) = 3$.

Calculus 1 — Exam 2 **MAT 250, Fall 2019 — D. Ivanšić**

Name: ______________________________

Show all your work!

Differentiate and simplify where appropriate:

1. (6pts) $\dfrac{d}{dx}\left(3x^7 - b^3 + \sqrt[5]{x^8} - \dfrac{7}{x^6}\right) =$

2. (6pts) $\dfrac{d}{dx}\left(x\sqrt{x+3}\right) =$

3. (6pts) $\dfrac{d}{dt}\dfrac{t^2-1}{2t+5} =$

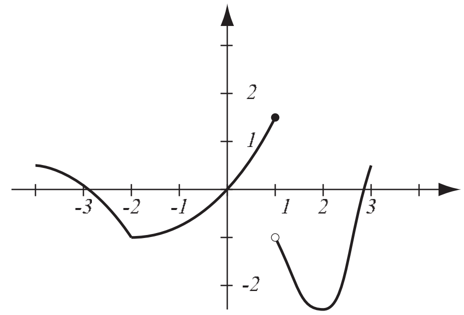
4. (7pts) $\dfrac{d}{d\theta}\dfrac{\sin\theta}{\cos^3\theta} =$

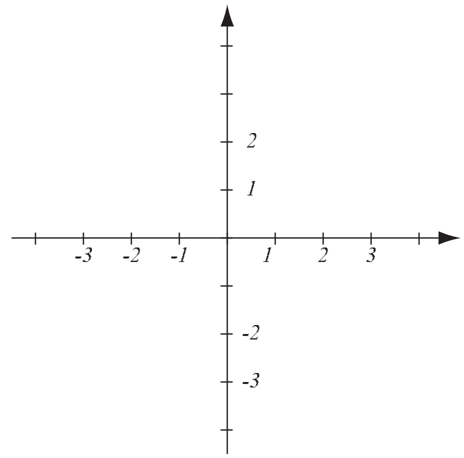
5. (6pts) $\dfrac{d}{dx}\sqrt[3]{\cos(x^2-7)} =$

6. (6pts) The position function of an object is given by $s(t) = t^2 - \sin(2t)$. Write the velocity and acceleration functions for this motion.

7. (10pts) The graph of the function $f(x)$ is shown at right.

a) Where is $f(x)$ not differentiable? Why?
b) Use the graph of $f(x)$ to draw an accurate graph of $f'(x)$.





8. (13pts) Let $f(x) = \sqrt{x}$, and $x > 0$.
a) Use the limit definition of the derivative to find the derivative of the function.
b) Check your answer by taking the derivative of f using differentiation rules.
c) Write the equation of the tangent line to the curve $y = f(x)$ at point $(9, 3)$.

9. (10pts) Let $g(x) = \dfrac{f(x)}{x^2}$ and $h(x) = f(x \cdot f(x))$.
a) Find the general expressions for $g'(x)$ and $h'(x)$.
b) Use the table of values at right to find $g'(3)$ and $h'(2)$.

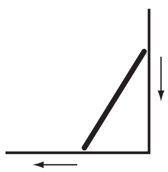
x	1	2	3	4
$f(x)$	-1	2	3	-5
$f'(x)$	-2	3	4	-1

10. (8pts) Find the point (x-coordinates only) on the curve $y = 2x^3 - 3x^2 - 31x + 7$ where the tangent line is parallel to the line $y = 5x - 17$.

11. (10pts) Use implicit differentiation to find y'.

$\sqrt{xy} = x^3 + y^3 - \tan y$

12. (12pts) A 9-foot ladder is sliding down the wall against which it is leaning. When the bottom of the ladder is 4 feet from the base of the wall, it is moving away from the wall at speed $\frac{1}{3}$ feet per second. How fast is the top of the ladder dropping at that moment?



Bonus. (10pts) Find points on the circle $x^2 + y^2 = 20$ where the tangent line is parallel to the line $6x - 2y = 4$. Draw the circle, the given line and the parallel tangent line(s). *(Hint: implicit differentiation is a little easier.)*

Calculus 1 — Exam 3
MAT 250, Fall 2019 — D. Ivanšić

Name: ______________________________

Show all your work!

Differentiate and simplify where appropriate:

1. (5pts) $\dfrac{d}{dx}\ln(3x^2-5x+2)=$

2. (6pts) $\dfrac{d}{dx}(x^{\frac{3}{2}}-4x^{\frac{1}{2}})e^x=$

3. (6pts) $\dfrac{d}{du}\dfrac{\ln u}{u^2}=$

4. (7pts) $\dfrac{d}{dx}\arctan\sqrt{x^2-1}=$

5. (7pts) $\dfrac{d}{d\theta}\log_4\dfrac{1-\sec\theta}{1+\sec\theta}=$

6. (10pts) Use logarithmic differentiation to find the derivative of $y=(x^2-1)^{x^2}$.

Find the limits algebraically. Graphs of basic functions will help, as will L'Hospital's rule, where appropriate.

7. (2pts) $\lim\limits_{x \to 0+} \log_3 x =$

8. (7pts) $\lim\limits_{x \to \infty} \arccos\left(\dfrac{x+4}{x^2 - 3x + 12}\right) =$

9. (6pts) $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x^2} =$

10. (9pts) $\lim\limits_{x \to \infty} \dfrac{x^{\frac{3}{2}}}{1.1^x} =$

11. (8pts) $\lim\limits_{x \to 0} (1 - 2x)^{\frac{1}{\sin x}} =$

12. (10pts) Let $f(x) = \sqrt{x}$.
a) Write the linearization of $f(x)$ at $a = 16$.
b) Use the linearization to estimate $\sqrt{17}$ and compare to the calculator value of 4.123106.

13. (10pts) A cube is measured to have side length of 5 centimeters, with maximum error 2 millimeters. Use differentials to estimate the maximum possible error, the relative error and the percentage error when computing the volume of the cube.

14. (7pts) The table of values of $f(x)$ and $f'(x)$ is given at right. Use the theorem on derivatives of inverses to find $(f^{-1})'(1)$.

x	1	2	3	4
$f(x)$	4	1	0	-1
$f'(x)$	-2	-3	-4	-1

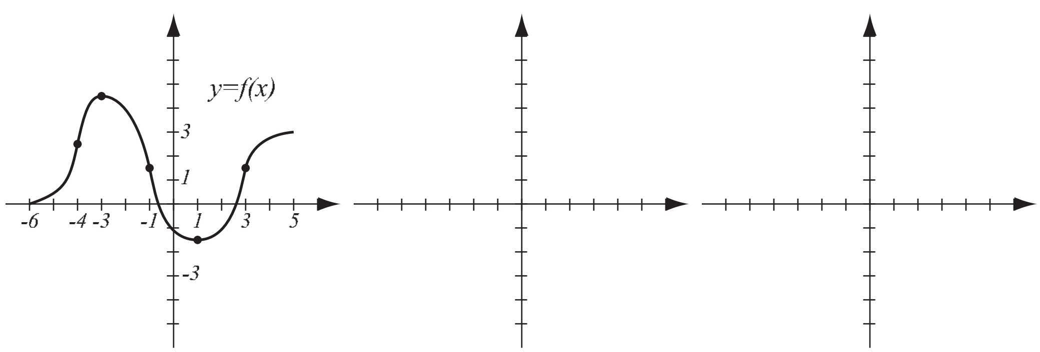
Bonus. (10pts) The function $f(x) = x^2 + 4x - 7$ is one-to-one on the domain $(-\infty, -2]$.
a) Use either the quadratic formula or completion of squares to find $f^{-1}(x)$.
b) Use the theorem on derivatives on inverses to find $(f^{-1})'(x)$ and compare it with the derivative that you get from the formula you find in a).

Calculus 1 — Exam 4 **MAT 250, Fall 2019 — D. Ivanšić**	**Name:** ____________________ *Show all your work!*

1. (30pts) Let $f(x) = x^3 e^x$. Draw an accurate graph of f by following the guidelines.
a) Find the intervals of increase and decrease, and local extremes.
b) Find the intervals of concavity and points of inflection.
c) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
d) Use information from a)–c) to sketch the graph.

2. (18pts) Let $f(x) = \sin^3 x + \cos^3 x$. Find the absolute minimum and maximum values of f on the interval $[0, \pi]$.

3. (14pts) The graph of f is given. Use it to draw the graphs of f' and f'' in the coordinate systems provided. Pay attention to increasingness, decreasingness and concavity of f. The relevant special points have been highlighted.

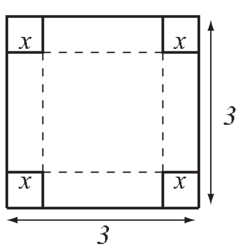


4. (16pts) Consider $f(x) = \dfrac{x+5}{x+1}$ on the interval $[0, 3]$.

a) Verify that the function satisfies the assumptions of the Mean Value Theorem.

b) Find all numbers c that satisfy the conclusion of the Mean Value Theorem.

5. (22pts) A 3×3 square piece of cardboard is to be made into a box by cutting out four smaller squares from the corners and folding the flaps upward. Dashed lines show where the cardboard is folded after the corner squares are removed. What size of removed squares produces the maximal possible volume of the resulting open-top box?



Bonus. (10pts) The Bernoulli inequality states that $(1+x)^n \geq 1+nx$ for every natural number n and every real number $x > -1$. Prove this inequality using calculus as follows:
a) Find the absolute minimum of the function $f(x) = (1+x)^n - nx$ on the interval $[-1, \infty)$.
b) Use the absolute minimum to conclude the inequality holds.

Calculus 1 — Exam 5
MAT 250, Fall 2019 — D. Ivanšić

Name: ____________________

Show all your work!

Find the following antiderivatives.

1. (3pts) $\displaystyle\int \sqrt[7]{x^3}\,dx =$

2. (3pts) $\displaystyle\int \frac{5}{\sqrt{1-x^2}}\,dx =$

3. (3pts) $\displaystyle\int e^{4x+1}\,dx =$

4. (7pts) $\displaystyle\int \frac{s^3+s}{\sqrt{s}}\,ds =$

5. (7pts) Find $f(x)$ if $f'(x) = \dfrac{3}{1+x^2} + \dfrac{1}{x}$ and $f(1) = 5$.

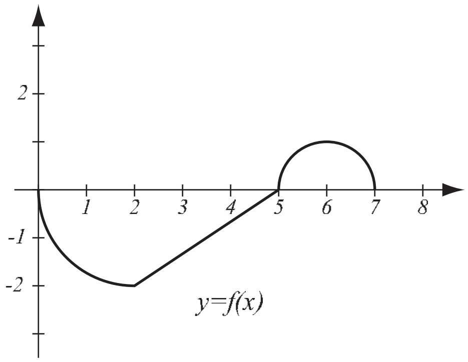
6. (6pts) Write using sigma notation:

$$\frac{5}{6} + \frac{7}{8} + \frac{9}{10} + \cdots + \frac{17}{18} =$$

7. (15pts) The function $f(x) = \sin x$ is given on the interval $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right]$.
a) Write the Riemann sum R_6 for this function with six subintervals, taking sample points to be right endpoints. Do not evaluate the expression.
b) Illustrate with a diagram, where appropriate rectangles are clearly visible. What does R_6 represent?

8. (13pts) Find $\int_{-2}^{2} 3 - x\, dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture.
b) Using the Evaluation Theorem.

9. (7pts) The graph of a function f, consisting of lines and parts of circles, is shown. Evaluate the integrals.



$\int_2^5 f(x)\,dx =$

$\int_5^7 f(x)\,dx =$

$\int_0^7 f(x)\,dx =$

Use the substitution rule in the following integrals:

10. (8pts) $\displaystyle\int \frac{3x^2+4x}{(x^3+2x^2+7)^2}\,dx =$

11. (10pts) $\displaystyle\int_0^{\ln 5} \frac{e^x}{\sqrt{4+e^x}}\,dx =$

12. (8pts) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (\sin^2 x - 3\sin x + 3)\cos x\,dx =$

13. (10pts) A ball traveling upwards has speed $v(t) = 27 - 10t$ meters per second..
a) Use the Net Change Theorem to find by how much the height of the ball has changed from $t = 0$ to $t = 3$.
b) If at time $t = 0$ the ball was at height 18 meters, at what height is it at $t = 3$?

Bonus. (10pts) Justify the following statements with pictures.
a) If f is even, then $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$
b) If f is odd, then $\int_{-a}^{a} f(x)\,dx = 0$

Calculus 1 — Final Exam MAT 250, Fall 2019 — D. Ivanšić

Name: ______________________________

Show all your work!

1. (15pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

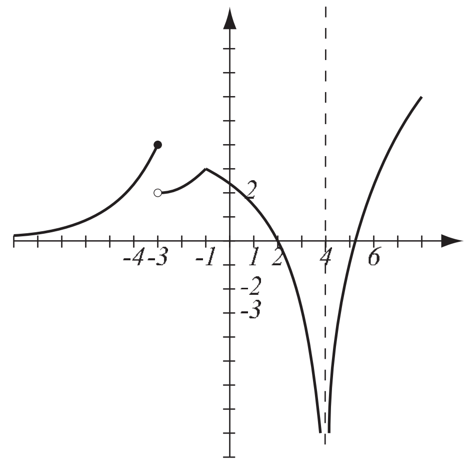
$\lim\limits_{x \to -3^-} f(x) =$

$\lim\limits_{x \to -3^+} f(x) =$

$\lim\limits_{x \to -3} f(x) =$

$\lim\limits_{x \to 4} f(x) =$

$\lim\limits_{x \to \infty} f(x) =$



List points where f is not continuous and explain why.

List points where f is not differentiable and explain why.

Find the following limits algebraically. Do not use L'Hospital's rule.

2. (6pts) $\lim\limits_{x \to 2^+} \dfrac{x^2+3}{2-x} =$

3. (5pts) $\lim\limits_{x \to -4} \dfrac{x^2-x-20}{x^2-16} =$

4. (6pts) The equation $x^2 + 1 = 2^x$ is given. Use the Intermediate Value Theorem to show it has a solution.

5. (9pts) Find the equation of the tangent line to the curve $y = x^2 + \frac{1}{x}$ at the point where $x = 2$.

6. (10pts) Let $f(x) = \sqrt{x}$.
a) Write the linearization of $f(x)$ at $a = 25$.
b) Use the linearization to estimate $\sqrt{27}$ and compare to the calculator value of 5.196152.

7. (26pts) Let $f(x) = \dfrac{x^2}{e^x}$. Draw an accurate graph of f by following the guidelines.

a) Find the intervals of increase and decrease, and local extremes.

b) Find the intervals of concavity and points of inflection.

c) Find $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$. Use L'Hospital's rule where necessary.

d) Use information from a)–c) to sketch the graph.

8. (10pts) Let $f(x) = x + 2\cos x$. Find the absolute minimum and maximum values of f on the interval $\left[0, \frac{\pi}{2}\right]$.

9. (6pts) Find $f(x)$ if $f'(x) = \sqrt[3]{x} + \sec x \tan x$ and $f(0) = 3$.

10. (11pts) Consider the integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \sin\theta \, d\theta$.

a) Use a picture and the "area" interpretation of the integral to determine whether this integral is positive or negative.

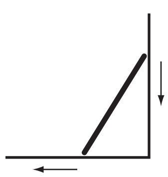
b) Use the Evaluation Theorem to find the integral and verify your conclusion from a).

Use the substitution rule in the following integrals:

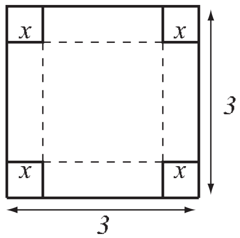
11. (7pts) $\displaystyle\int \frac{1}{(1+(\ln x)^2)\,x}\,dx =$

12. (9pts) $\displaystyle\int_{-1}^{1} (3x^2-4x)e^{x^3-2x^2+1}\,dx =$

13. (12pts) An 8-foot ladder is sliding down the wall against which it is leaning. When the bottom of the ladder is 5 feet from the base of the wall, it is moving away from the wall at speed $\frac{1}{2}$ feet per second. How fast is the top of the ladder dropping at that moment?



14. (18pts) A 3×3 square piece of cardboard is to be made into a box by cutting out four smaller squares from the corners and folding the flaps upward. Dashed lines show where the cardboard is folded after the corner squares are removed. What size of removed squares produces the maximal possible volume of the resulting open-top box?



Bonus. (10pts) Find points on the circle $x^2 + y^2 = 20$ where the tangent line is parallel to the line $6x - 2y = 4$. Draw the circle, the given line and the parallel tangent line(s). *(Hint: implicit differentiation is a little easier.)*