Calculus 1 — Exam 5
MAT 250, Spring 2017 — D. Ivanšić

Name: _____

Show all your work!

Find the following antiderivatives.

1. (3pts) $\displaystyle\int \frac{1}{\sqrt[4]{x^3}}\,dx =$

2. (3pts) $\displaystyle\int \frac{4}{1+x^2}\,dx =$

3. (3pts) $\displaystyle\int \cos\left(2x+\frac{\pi}{2}\right)\,dx =$

4. (7pts) $\displaystyle\int (t^3-4t^2)\sqrt{t}\,dt =$

5. (7pts) Find $f(x)$ if $f'(x) = 2e^{4x} + \sec x \tan x$ and $f(0) = 3$.

6. (6pts) Write using sigma notation:

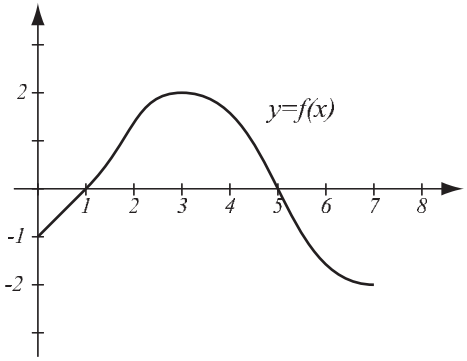
$-3 + 6 - 9 + 12 - 15 + \cdots + 300 - 303 =$

7. (15pts) The function $f(x) = 2^x$ is given on the interval $[-1, 1]$.
a) Write the Riemann sum M_6 for this function with six subintervals, taking sample points to be midpoints. Do not evaluate the expression.
b) Illustrate with a diagram, where appropriate rectangles are clearly visible. What does M_6 represent?

8. (13pts) Find $\int_0^2 2x - 1\, dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture.
b) Using the Evaluation Theorem.

9. (7pts) The graph of a function f is shown. Put the four numbers 0, a, b, c in increasing order and justify your reasoning.

$a = \int_0^1 f(x)\,dx$ $\qquad$ $b = \int_0^5 f(x)\,dx$ $\qquad$ $c = \int_0^7 f(x)\,dx$



Use the substitution rule in the following integrals:

10. (8pts) $\displaystyle\int (x+2)\sqrt[9]{x^2+4x+3}\,dx =$

11. (10pts) $\displaystyle\int_1^{e^\pi} \frac{\sin(\ln x)}{x}\,dx =$

12. (8pts) $\displaystyle\int_2^4 \frac{2x-6}{\cot(x^2-6x+5)}\,dx =$

13. (10pts) The rate at which a river's water level is changing is $t^2 - 4t - 21$ feet per day.
a) Use the Net Change Theorem to find by how much the river water level has changed from $t = 6$ to $t = 9$.
b) If at time $t = 6$ the river water level was 32 feet, what is it at time $t = 9$?

Bonus. (10pts) Evaluate. A picture will help.

$$\int_{\frac{\pi}{6}}^{\frac{4\pi}{3}} |\sin x|\, dx =$$