Calculus 1 — Exam 4
MAT 250, Spring 2017 — D. Ivanšić

Name: ____________________

Show all your work!

1. (30pts) Let $f(x) = (x^2 + x + 2)e^x$. Draw an accurate graph of f by following the guidelines.
a) Find the intervals of increase and decrease, and local extremes.
b) Find the intervals of concavity and points of inflection.
c) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
d) Use information from a)–c) to sketch the graph.

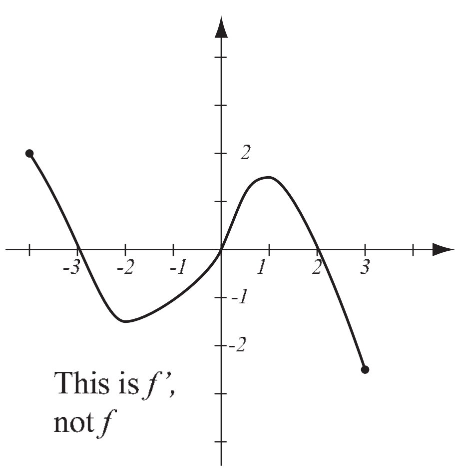
2. (14pts) Let $f(x) = \sin^2 x - \cos x$. Find the absolute minimum and maximum values of f on the interval $[0, \pi]$.

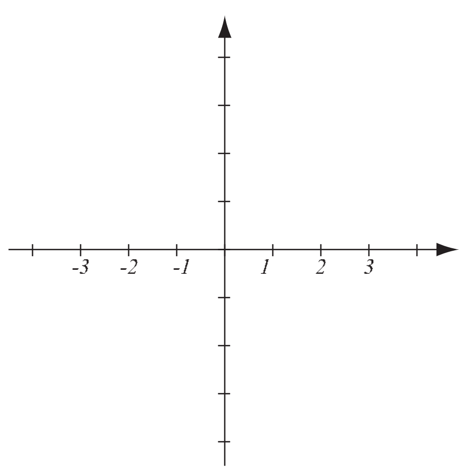
3. (18pts) Let f be continuous on $[-4, 3]$. The graph of its derivative f' is drawn below. Use the graph to answer (sign charts may help):

a) What are the intervals of increase and decrease of f? Where does f have a local minimum or maximum?

b) What are the intervals of concavity of f? Where does f have inflection points?

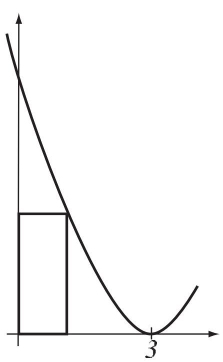
c) Use the information gathered in a) and b) to sketch the graph of f at right, if $f(-4) = 0$.





4. (16pts) Consider $f(x) = x^2 - 3x + 5$ on the interval $[1, 4]$.
a) Verify that the function satisfies the assumptions of the Mean Value Theorem.
b) Find all numbers c that satisfy the conclusion of the Mean Value Theorem.

5. (22pts) Consider a rectangle with sides on the x- and y-axes whose one vertex lies on the parabola $y = (x-3)^2$ and is enclosed in the region between the axes and the parabola. Among all such rectangles, find the one with the biggest area.



Bonus. (10pts) Draw a function, if possible, that satisfies the given conditions. Justify if such a function is not possible.

a) f defined on $[1, 4]$, has a local maximum but no absolute maximum.

b) f continuous on $[1, 4]$, has a local minimum but no absolute maximum.

c) f defined on $[1, 4)$, has no local minimum nor maximum, and has no absolute minimum nor maximum.