Calculus 1 — Exam 1
MAT 250, Fall 2019 — D. Ivanšić

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Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to -2+} f(x) = 4$

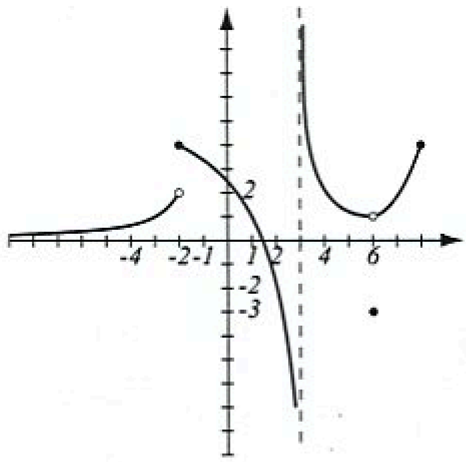
$\lim_{x\to -2^-} f(x) = 2$

$\lim_{x\to -2} f(x) =$ dne (left and right limits are different)

$\lim_{x\to -\infty} f(x) = 0$

$\lim_{x\to 6} f(x) = 1$

$\lim_{x\to 3} f(x) =$ dne (left and right limits are different)



List points in domain of f where f is not continuous and justify why it is not continuous at those points.

Not continuous at $x=-2$: $\lim_{x\to -2} f(x)$ d.n.e,

$x=6$: $\lim_{x\to 6} f(x) = 1$, $f(6)=-3$ different values

2. (6pts) Let $\lim_{x\to 5} f(x) = 3$ and $\lim_{x\to 5} g(x) = 1$. Use limit laws to find the limit below and show each step.

$$\lim_{x\to 5} \frac{xf(x)-9g(x)}{f(x)^2-3} = \frac{\lim_{x\to 5}(xf(x)-9g(x))}{\lim_{x\to 5}(f(x)^2-3)} = \frac{\lim_{x\to 5}(xf(x)) - \lim_{x\to 5} 9g(x)}{\lim_{x\to 5} f(x)^2 - \lim_{x\to 5} 3}$$

$$= \frac{\lim_{x\to 5} x \cdot \lim_{x\to 5} f(x) - 9\lim_{x\to 5} g(x)}{\left(\lim_{x\to 5} f(x)\right)^2 - \lim_{x\to 5} 3} = \frac{5\cdot 3 - 9\cdot 1}{3^2-3} = \frac{6}{6} = 1$$

3. (10pts) Find $\lim_{x\to 0} x^4 \cdot (1+\sin\left(\frac{1}{x}\right))^2$. Use the theorem that rhymes with insects that you might find on dogs and cats.

$-1 \le \sin\frac{1}{x} \le 1$

$0 \le 1+\sin\frac{1}{x} \le 2$

$0 \le \left(1+\sin\frac{1}{x}\right)^2 \le 4$

$0 \le x^4\left(1+\sin\frac{1}{x}\right)^2 \le 4x^4$

$\lim_{x\to 0} 0 = 0$, $\lim_{x\to 0} 4x^4 = 0$ } same, so by the squeeze theorem, we have

$$\lim_{x\to 0} x^4\left(1+\sin\frac{1}{x}\right)^2 = 0$$

Find the following limits algebraically. Do not use the calculator.

4. (7pts) $\lim_{x\to\infty} \frac{x^2-5x+7}{x+4} = \lim_{x\to\infty} \frac{\cancel{x}^2\left(1-\frac{5}{x}+\frac{7}{x^2}\right)}{\cancel{x}\left(1+\frac{4}{x}\right)} = \infty \cdot \frac{1-0+0}{1+0} = \infty \cdot 1 = \infty$

$\to 0 \quad \to 0 \quad \to 0$

$\frac{\infty}{\infty}$

$\frac{0}{0}$

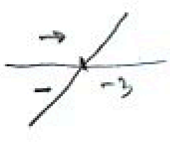
5. (5pts) $\lim_{x\to 7} \frac{x^2-6x-7}{x^2-11x+28} = \lim_{x\to 7} \frac{\cancel{(x-7)}(x+1)}{\cancel{(x-7)}(x-4)} = \frac{7+1}{7-4} = \frac{8}{3}$

$\frac{0}{0}$

6. (7pts) $\lim_{x\to 5} \frac{x^2-25}{\sqrt{x}-\sqrt{5}} = \lim_{x\to 5} \underbrace{\frac{x^2-25}{\sqrt{x}-\sqrt{5}} \cdot \frac{\sqrt{x}+\sqrt{5}}{\sqrt{x}+\sqrt{5}}}_{=\sqrt{x}^2-\sqrt{5}^2} = \lim_{x\to 5} \frac{\cancel{(x-5)}(x+5)(\sqrt{x}+\sqrt{5})}{\cancel{x-5}\ 1}$

$= \frac{(5+5)(\sqrt{5}+\sqrt{5})}{1} = 20\sqrt{5}$

7. (6pts) $\lim_{x\to -3^-} \frac{x}{x+3} = \frac{-3}{0^-} = -3 \cdot \frac{1}{0^-} = -3 \cdot (-\infty) = \infty$



$\frac{1}{0} - \frac{1}{0}$

8. (7pts) $\lim_{x\to 1^+} \left(\frac{1}{x-1} - \frac{2}{x^2-1}\right) = \lim_{x\to 1^+} \left(\frac{1}{x-1} - \frac{2}{(x-1)(x+1)}\right)$

$= \lim_{x\to 1^+} \frac{x+1-2}{(x-1)(x+1)} = \lim_{x\to 1^+} \frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)} = \frac{1}{1+1} = \frac{1}{2}$

$x^3 - x^2 + x - \sqrt{x} = 1$

9. (8pts) The equation $x^3 - x^2 + x = \sqrt{x} + 1$ is given. Use the Intermediate Value Theorem to show it has a solution in the interval $(0, 4)$.

Let $f(x) = x^3 - x^2 + x - \sqrt{x}$

it is continuous.

$f(0) = 0$

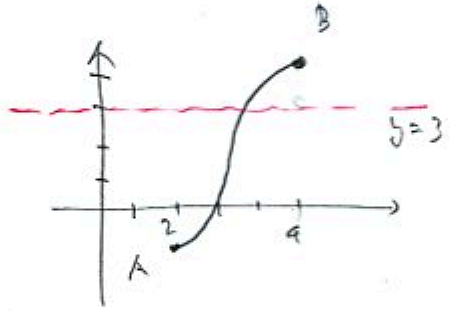
$f(4) = 4^3 - 4^2 + 4 - \sqrt{4} = 50$

$= 64 - 16 + 4 - 2$

$f(0) \quad f(4)$

Since $0 < 1 < 50$, by IVT there is a number c in $(0, 4)$ such that $f(c) = 1$, so a solution to the equation

10. (8pts) Explain in an intuitive way why the Intermediate Value Theorem is true on this example: Let f be a continuous function defined on the interval $[2, 5]$, and let $f(2) = -1$ and $f(5) = 4$. Justify graphically why there has to be a number c in the interval $(2, 5)$ so that $f(c) = 3$. (You need a picture and a nice sentence.)



An unbroken curve from A to B has to cross the line $y = 3$ somewhere between 2 and 4

11. (10pts) Consider the limit $\lim_{x \to 2} \frac{2^x - 4}{x - 2}$. Use your calculator (don't forget parentheses) to estimate this limit with accuracy 3 decimal points. Write a table of values that will justify your answer.

x	$\frac{2^x - 4}{x - 2}$
2.1	2.870939
2.01	2.782220
2.001	2.773549
2.0001	2.772684
2.00001	2.772598

x	$\frac{2^x - 4}{x - 2}$
1.9	2.678680
1.99	2.763002
1.999	2.771628
1.9999	2.772493
1.99999	2.772579

It appears

$\lim_{x \to 2} \frac{2^x - 4}{x - 2} = 2.772$

to three decimals

12. (10pts) Is the function defined below continuous? Justify.

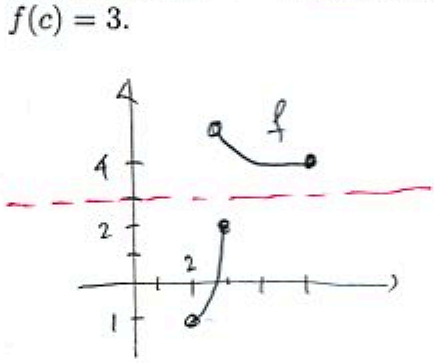
$$f(x) = \begin{cases} \dfrac{\sin(2x-6)}{x-3}, & \text{if } x \neq 3 \\ 1 & \text{if } x = 3. \end{cases}$$

$\frac{0}{0}$

$$\lim_{x\to 3} \frac{\sin(2x-6)}{x-3} = \lim_{x\to 3} \frac{\sin(2x-6)}{2(x-3)} \cdot 2 = \lim_{x\to 3} \underbrace{\frac{\sin(2x-6)}{2x-6}}_{=1} \cdot 2 = 1 \cdot 2 = 2$$

$f(3) = 1$ so $\lim_{x\to 2} f(x) \neq f(2)$ like $\lim_{h\to 0} \frac{\sin h}{h} = 1$

Bonus. (10pts) Show by example that the conclusion of the Intermediate Value Theorem is not true if the function is not continuous. Draw a function defined on the interval $[2,5]$ for which $f(2) = -1$ and $f(5) = 4$. but there is no number c in the interval $(2,5)$ so that $f(c) = 3$.



Example cannot be continuous, or it would satisfy the IVT.

There is no c for which $f(c) = 3$ since the line $y = 3$ does not intersect the graph.