

1. (5pts) If  $\log_a 4 = u$  and  $\log_a 5 = v$ , express in terms of  $u$  and  $v$ :

$$\begin{aligned}\log_a 20 &= \log_a (4 \cdot 5) \\ &= \log_a 4 + \log_a 5 \\ &= u + v\end{aligned}$$

$$\begin{aligned}\log_a \frac{25}{4} &= \log_a 25 - \log_a 4 \\ &= \log_a 5^2 - u \\ &= 2 \log_a 5 - u \\ &= 2v - u\end{aligned}$$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\begin{aligned}\log_5 (25x^{-7}y^3) &= \log_5 25 + \log_5 x^{-7} + \log_5 y^3 \\ &= 2 - 7 \log_5 x + 3 \log_5 y\end{aligned}$$

$$\begin{aligned}\log_2 \frac{x^{\frac{5}{2}} y^3}{32 \sqrt{xy^5}} &= \log_2 x^{\frac{5}{2}} + \log_2 y^3 - \log_2 32 - \log_2 x^{\frac{1}{2}} - \log_2 y^5 \\ &\quad \uparrow \\ &\quad x^{\frac{1}{2}} \\ &= \frac{5}{2} \log_2 x + 3 \log_2 y - 5 - \frac{1}{2} \log_2 x - 5 \log_2 y \\ &= 2 \log_2 x - 2 \log_2 y - 5\end{aligned}$$

3. (12pts) Write as a single logarithm. Simplify if possible.

$$\begin{aligned}2 \log(3x^4) - \frac{1}{3} \log(27y^6) + 5 \log x &= \log (3x^4)^2 - \log (27y^6)^{\frac{1}{3}} + \log x^5 \\ &= \log \frac{3^2 x^8 \cdot x^5}{3 \cdot y^2} = \log \frac{3x^{13}}{y^2}\end{aligned}$$

$27^{\frac{1}{3}} = 3\sqrt[3]{27}$

$$\begin{aligned}2 \ln(x^2 - 4x - 5) - 3 \ln(x - 5) - 2 \ln(x + 1) &= \ln \left( (x-5)(x+1) \right)^2 - \ln (x-5)^3 - \ln (x+1)^2 \\ &= \ln \frac{(x-5)^2 (x+1)^2}{(x-5)^3 (x+1)^2} = \ln \frac{1}{x-5} = -\ln(x-5)\end{aligned}$$

$(x-5)(x+1)$

Solve the equations.

4. (5pts)  $6^{2x-3} = \left(\frac{1}{36}\right)^{x+1}$

$$6^{2x-3} = (6^{-2})^{x+1}$$

$$6^{2x-3} = 6^{-2x-2}$$

$$2x-3 = -2x-2$$

$$4x = 1$$

$$x = \frac{1}{4}$$

6. (8pts)  $e^{2x} + 11 \cdot e^x - 26 = 0$

$$(e^x)^2 + 11e^x - 26 = 0$$

let  $u = e^x$

$$u^2 + 11u - 26 = 0$$

$$(u+13)(u-2) = 0$$

$$u = -13, 2$$

$e^x = -13$   
not possible  
since  $e^x > 0$

$$e^x = 2$$

$$x = \ln 2$$

5. (7pts)  $7^{x-4} = 4^{2x+1} \quad | \ln$

$$\ln 7^{x-4} = \ln 4^{2x+1}$$

$$(x-4) \ln 7 = (2x+1) \ln 4$$

$$x \ln 7 - 4 \ln 7 = 2 \ln 4 \cdot x + \ln 4$$

$$-4 \ln 7 - \ln 4 = 2 \ln 4 \cdot x - \ln 7 \cdot x$$

$$-4 \ln 7 - \ln 4 = (2 \ln 4 - \ln 7) x$$

$$x = -\frac{4 \ln 7 + \ln 4}{2 \ln 4 - \ln 7} = \frac{4 \ln 7 + \ln 4}{\ln 7 - 2 \ln 4}$$

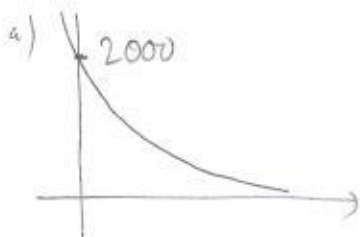
$$= -11.092503$$

7. (12pts) A car repair shop bought a car-lift for \$2000. Its value after  $t$  years is given by the function  $V(t) = 2000 \cdot 0.92^t$ .

a) Graph the value function.

b) What is the value after 3, 5 and 7 years?

c) When will the value of the car-lift be \$400?



b)

$t$	$2000 \cdot 0.92^t$
3	1557.38
5	1318.16
7	1115.69

c)  $400 = 2000 \cdot 0.92^t \quad | \div 2000$

$$0.2 = 0.92^t \quad | \ln$$

$$\ln 0.2 = \ln 0.92^t$$

$$\ln 0.2 = t \cdot \ln 0.92$$

$$t = \frac{\ln 0.2}{\ln 0.92} = 19.302673$$

is about 19 years, value is \$400