College Algebra — Joysheet 8	Name: Saul Ocean
MAT 140, Fall 2019 — D. Ivanšić	*Covers: 3.1, 3.2, 3.3 Show all your work!*

Simplify, so that the answer is in form $a+bi$.

1. (4pts) $(2+3i)(1-i)-i(4-5i)=$ $2+3i-2i-3i^2-(4i-5i^2)$

$= \underbrace{5+i-4i+5i^2}_{=-5} = -3i$

2. (6pts) $\frac{4+i}{2-3i}=$ $\frac{4+i}{2-3i}\cdot\frac{2+3i}{2+3i} = \frac{8+2i+12i+3i^2}{2^2-(3i)^2} = \frac{5+14i}{4-(-9)}$

$= \frac{5+14i}{13} = \frac{5}{13}+\frac{14}{13}i$

3. (4pts) Simplify and justify your answer.

$i^{85} =$ $i^{84}\cdot i^1 = \underbrace{(i^4)^{21}}_{=1}\cdot i^1 = 1\cdot i = i$

4. (8pts) The water level of a river above normal (in feet) is given by $H(x)=-x^2+14x-9$, where x is the number of days after October 28th.
a) On what dates was the water level 24 feet above normal?
b) On what date did the water level peak?

a) $-x^2+14x-9=24$

$-x^2+14x-33=0$

$x^2-14x+33=0$

$(x-11)(x-3)=0$

$x=3, 11$ days after Oct 28th

Dates are: Oct 31st, Nov 8th

b) Need vertex

$h=-\frac{b}{2a}=-\frac{14}{2\cdot(-1)}=7$

7 days after Oct 28th is Nov 4th

5. (8pts) Solve the equation: $x^6-3x^3-40=0$

$(x^3)^2-3x^3-40=0$ quadratic in form

let $u=x^3$

$u^2-3u-40=0$

$(u-8)(u+5)=0$

$u=8,-5$

$x^3=8$ or $x^3=-5$

$x=2$ or $x=-\sqrt[3]{5}$

6. (6pts) Solve by completing the square.

$x^2-\underbrace{12x}_{\downarrow}+8=0$ $\quad |+6^2$

$x^2-2\cdot x\cdot 6+6^2+8=6^2$

$(x-6)^2=36-8$

$(x-6)^2=28$

$x-6=\pm\sqrt{28}$

$4\cdot 7$

$x=6\pm 2\sqrt{7}$

7. (12pts) The quadratic function $f(x) = -x^2 + 2x - 7$ is given. Do the following without using the calculator.
a) Find the x-intercepts of its graph, if any. Find the y-intercept.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

a) $-x^2+2x-7=0$ x-int

$x^2-2x+7=0$

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4\cdot1\cdot7}}{2}$$

y-int,
$f(0)=-7$

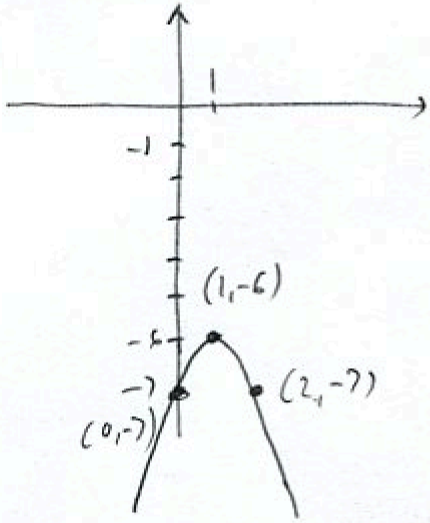
$$=\frac{2\pm\sqrt{-24}}{2}$$

no real sol, so no x-int

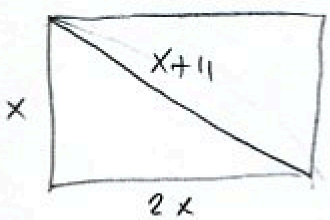
b) $h=-\frac{b}{2a}=-\frac{2}{2\cdot(-1)}=1$

$k=f(1)=-1+2-7=-6$

c)



8. (12pts) In a rectangle, one side is twice as long as another side, and the diagonal is 11 meters longer than the shorter side. What are the dimensions of the rectangle?



$x^2+(2x)^2=(x+11)^2$

$x^2+4x^2=x^2+2\cdot x\cdot 11+11^2$

$5x^2=x^2+22x+121$

$4x^2-22x-121=0$

$$x=\frac{-(-22)\pm\sqrt{(-22)^2-4\cdot4\cdot(-121)}}{2\cdot4}$$

$$=\frac{22\pm\sqrt{484+1936}}{8}$$

4.605

$$=\frac{22\pm\sqrt{2420}}{8}=\frac{22\pm2\sqrt{605}}{8}$$

$5\cdot11^2$

$$=\frac{\cancel{2}(11\pm\sqrt{605})}{\cancel{8}\,4}=\frac{11\pm\sqrt{605}}{4}=\frac{11\pm11\sqrt5}{4}$$

Since $\frac{11-11\sqrt5}{4}$ is negative, it's not possible for x $(x\geq0)$

Thus, $x=\frac{11+11\sqrt5}{4}$

Dimensions: $\frac{11+11\sqrt5}{4}$ by $\frac{11+11\sqrt5}{2}$