1. (21pts) For the following functions:
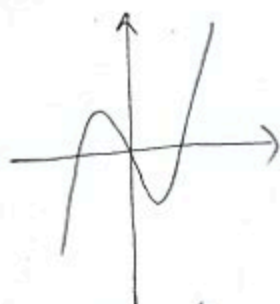a) determine algebraically whether they are odd, even, or neither
b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.
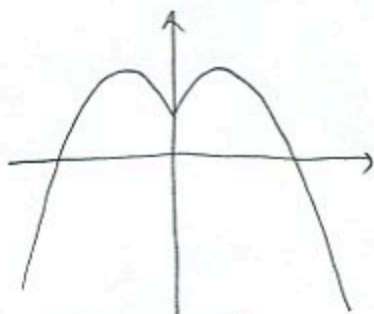
$f(x) = x^3 - 8x$

$f(-x) = (-x)^3 - 8(-x)$

$= -x^3 + 8x$

$= -f(x)$ odd

$g(x) = -x^2 + 7|x| + 3$

$g(-x) = -(-x)^2 + 7|-x| + 3$

$= -x^2 + 7|x| + 3$

$= g(x)$ even

$h(x) = x^4 + 2x^2 - 5x$

$h(-x) = (-x)^4 + 2(-x)^2 - 5(-x)$

$= x^4 + 2x^2 + 5x \neq h(x)$
$\neq -h(x)$

neither

symm. wrt origin

symm. wrt. y-axis
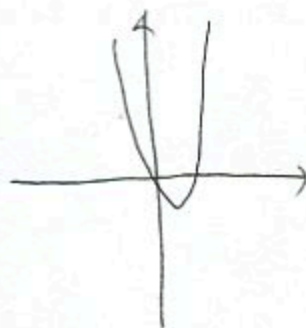
no symmetry

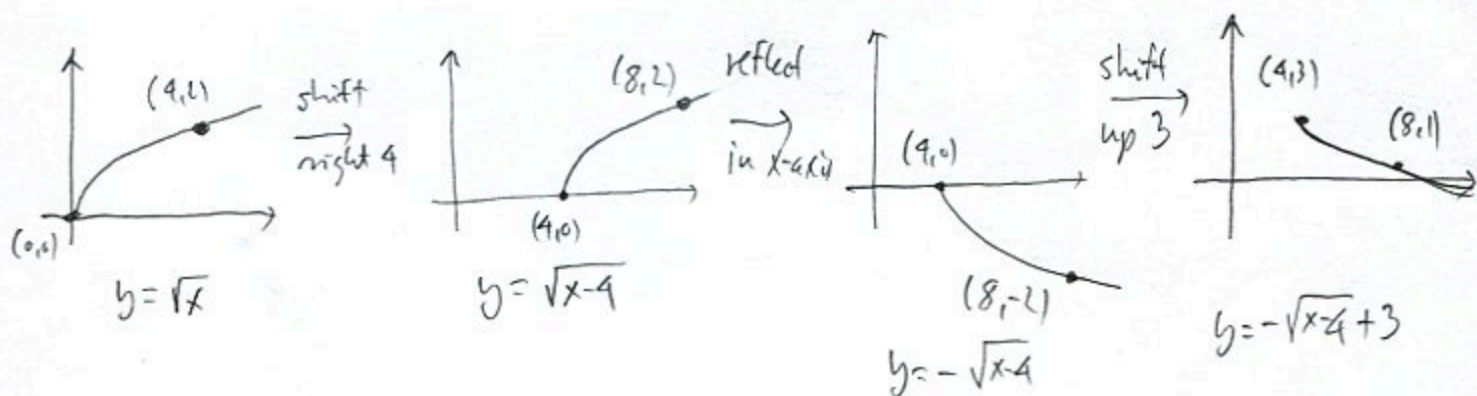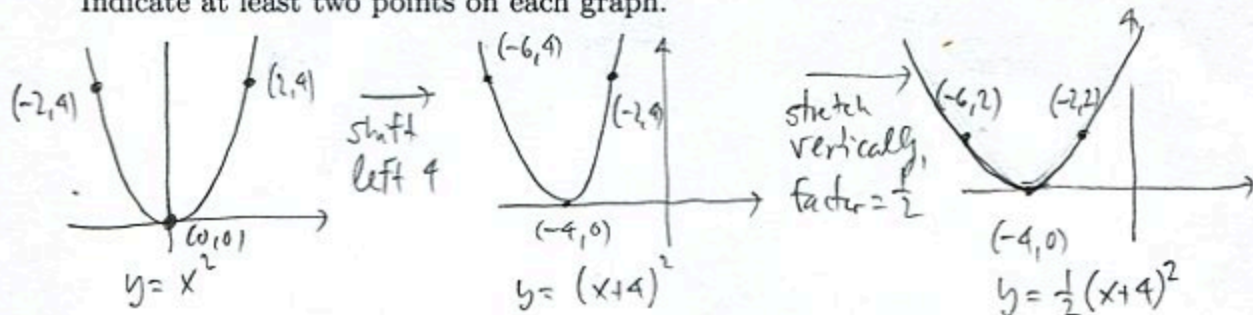**2.** (16pts) Draw the graphs of $f(x) = \frac{1}{2}(x+4)^2$ and $g(x) = 3 - \sqrt{x-4}$ using transformations. Explain how you transform graphs of basic functions in order to get the graphs of f and g. Indicate at least two points on each graph.

$(-2,4)$ $(2,4)$

$(0,0)$

$y = x^2$

shift left 4

$(-6,4)$ $(-2,4)$

$(-4,0)$

$y = (x+4)^2$

stretch vertically, factor $= \frac{1}{2}$

$(-6,2)$ $(-2,2)$

$(-4,0)$

$y = \frac{1}{2}(x+4)^2$

$(4,2)$

$(0,0)$

$y = \sqrt{x}$

shift right 4

$(8,2)$

$(4,0)$

$y = \sqrt{x-4}$

rfled in x-axis

$(4,0)$

$(8,-2)$

$y = -\sqrt{x-4}$

shift up 3

$(4,3)$

$(8,1)$

$y = -\sqrt{x-4} + 3$

3. (10pts) Write the equation for the function whose graph has the following characteristics:

a) shape of $y = \frac{1}{x}$, shifted right 5 units

b) shape of $y = x^2$ stretched horizontally by factor 2, then shifted down 1 unit

c) shape of $y = \sqrt[3]{x}$, reflected about the y-axis, then stretched vertically by factor 4, then shifted left 2 units.

a) $\dfrac{1}{x} \xrightarrow[\substack{x \text{ by } x-5}]{\text{replace}} \dfrac{1}{x-5}$

b) $x^2 \xrightarrow[\substack{x \text{ by } \frac{1}{2}x}]{\text{replace}} \left(\frac{1}{2}x\right)^2 \xrightarrow[\text{subtract } 1]{} \left(\frac{1}{2}x\right)^2 - 1 = \dfrac{x^2}{4} - 1$

c) $\sqrt[3]{x} \xrightarrow[\substack{x \text{ by } -x}]{\text{replace}} \sqrt[3]{-x} \xrightarrow[\substack{\text{multiply} \\ \text{by } 4}]{} 4\sqrt[3]{-x} \xrightarrow[\substack{x \text{ by } x+2}]{\text{replace}} 4\sqrt[3]{-(x+2)} = 4\sqrt[3]{-x-2}$

4. (13pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x) - 3$, $-f(\frac{1}{2}x)$ and $2f(x + 1)$ and label all the relevant points.

(-4,2)   2   (3,1)

-6   -3   -1   5

(1,-1)   -3

shift down 3

$y \mapsto y-3$

(-4/7,-1)   (8,-2)   (3,-2)

(1,-4)
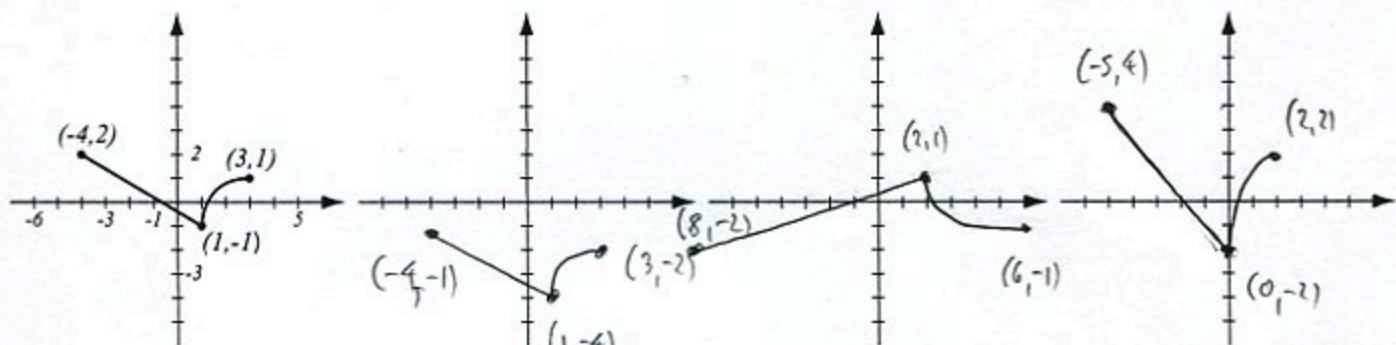
stretch horizontally,

Factor = 2

reflect in y axis

$x \mapsto 2x$

$y \mapsto -y$

(2,1)

(6,-1)

(-5,4)   (2,2)

(0,-2)

shift left 1

stretch vertically,

Factor = 2

$x \mapsto x-1$

$y \mapsto 2y$