College Algebra — Joysheet 5
MAT 140, Fall 2019 — D. Ivanšić

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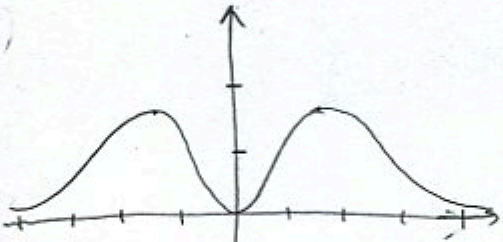
Covers: 2.1, 2.2, 2.3 Show all your work!

1. (10pts) Use your calculator to accurately sketch the graph of the function $f(x) = \dfrac{5x^2+1}{x^4+3}$. Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.

a) Find the local maxima and minima for this function.

b) State the intervals where the function is increasing and where it is decreasing.



a) $\left.\begin{array}{l} f(1.2424) = 1.619633 \\ f(-1.2424) = 1.619633 \end{array}\right\}$ are local maxima

$f(0) = 0.333333$ is a local minimum

b) Increasing on $(-\infty, -1.2424)$ and $(0, 1.2424)$

Decreasing on $(-1.2424, 0)$ and $(1.2424, \infty)$

2. (20pts) Let $f(x) = \dfrac{x^2}{\sqrt{3x+9}}$, $g(x) = \sqrt{17-2x}$. Find the following (simplify where possible):

$(f+g)(-1) = f(-1)+g(-1) = \dfrac{(-1)^2}{\sqrt{3\cdot(-1)+9}} + \sqrt{17-2\cdot(-1)}$

$= \dfrac{1}{\sqrt{6}} + \sqrt{19}$

$(fg)(4) = f(4)\cdot g(4) = \dfrac{4^2}{\sqrt{3\cdot 4+9}} \cdot \sqrt{17-2\cdot 4}$

$= \dfrac{16}{\sqrt{21}} \cdot \sqrt{9} = \dfrac{48}{\sqrt{21}}$

$\dfrac{g}{f}(x) = \dfrac{g(x)}{f(x)} = \dfrac{\sqrt{17-2x}}{\dfrac{x^2}{\sqrt{3x+9}}} =$

$= \dfrac{\sqrt{17-2x}}{1} \cdot \dfrac{\sqrt{3x+9}}{x^2} = \dfrac{\sqrt{(17-2x)(3x+9)}}{x^2}$

$(g \circ f)(9) = g(f(9)) = g\left(\dfrac{9^2}{\sqrt{3\cdot 9+9}}\right) = g\left(\dfrac{81}{\sqrt{36}}\right)$

$= g\left(\dfrac{81}{6}\right) = g\left(\dfrac{27}{2}\right) = \sqrt{17-2\cdot\dfrac{27}{2}}$

$= \sqrt{-10}$ not defined

$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{17-2x}\right)$

$= \dfrac{\left(\sqrt{17-2x}\right)^2}{\sqrt{3\sqrt{17-2x}+9}} = \dfrac{17-2x}{\sqrt{3\sqrt{17-2x}+9}}$

The domain of $(fg)(x)$ in interval notation

Domain is $\left(-3, \dfrac{17}{2}\right]$

domain of f: must have $3x+9>0$

$3x > -9$

$x > -3$

domain of g: must have

$17-2x \geq 0$

$-2x \geq -17$

$x \leq \dfrac{17}{2}$

f

g

overlap

-3 $\quad \dfrac{17}{2}$

3. (8pts) Consider the function $h(x) = \dfrac{8}{x^2 - 3}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

$g(x) = x^2$ $\qquad\qquad$ $g(x) = x^2 - 3$

$f(x) = \dfrac{8}{x-3}$ $\qquad\qquad$ $f(x) = \dfrac{8}{x}$

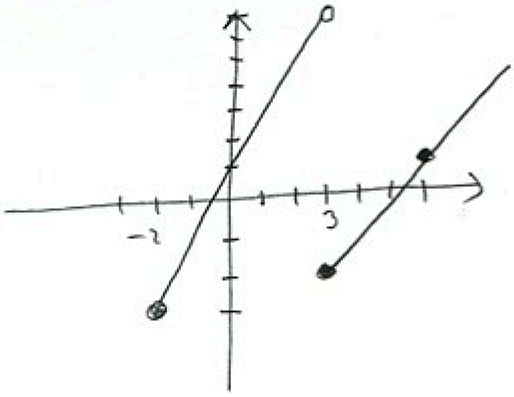
4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x+1, & \text{if } -2 \le x < 3 \\ x-5, & \text{if } x \ge 3. \end{cases}$$

(parts of lines)

x	$2x+1$
-2	-3
3	7

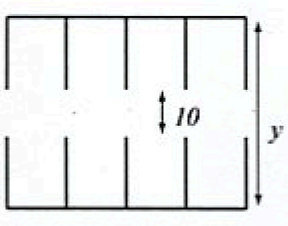
x	$x-5$
3	-2
6	1



5. (14pts) Rodrigo is building a stable with area 1100 square feet and eight stalls with a gap between them of 10 feet. He wishes to minimize the building cost, which is the same as minimizing the total length of the walls.

a) Express the total length of the walls of the building as a function of the length of one of the sides x. What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the stable for which the total length of the walls is minimal? What is the minimal wall length?



Total wall length $= \ell$

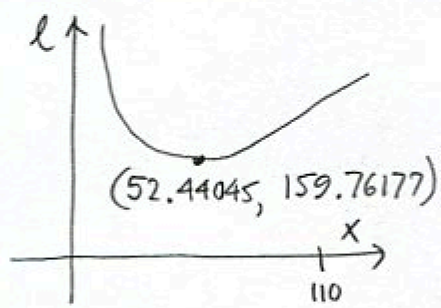
$\ell = 2x + 5(y-10)$

$= 2x + 5y - 50$

$= 2x + \dfrac{5500}{x} - 50$

$xy = 1100$ put in for y

$y = \dfrac{1100}{x}$



Must have:

$x > 0$

$y \geq 10$

$\dfrac{1100}{x} \geq 10$

$1100 \geq 10x$

$110 \geq x$

Domain:

$(0, 110]$

Dimensions $x \times y$ are

$52.44045 \times 20.976174$

Minimal wall length is

159.76177 ft