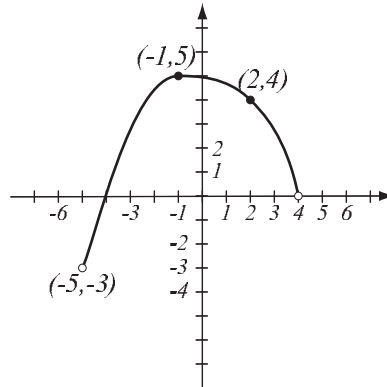**1.** (8pts) Use the graph of the function f
at right to answer the following questions.

a) Find: $f(-1) =$ $f(6) =$

b) What is the domain of f?

c) What is the range of f?

d) What are the solutions
   of the equation $f(x) = 4$?

2. (10pts) Use your calculator to accurately
sketch the graph of $y = x^3 - 4x^2 + x + 15$.

a) Draw the graph on paper and indicate units
on the axes.

b) Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).

3. (5pts) A line contains the point $(1, -2)$. If you start at any point on the line, go right 3
units and then down 2 units, you wind up back on the line. Write the equation of the line.

4. (10pts) Find the equation of the line (in form $y = mx + b$) that is perpendicular to the
line $3x - y = 2$ and passes through $(6, -3)$. Draw both lines.

5. (7pts) Draw the points $A = (3,4)$, $B = (-1,5)$ and $C = (-2,0)$ in the coordinate plane. Use the distance formula to determine which of A and B is closer to C.

6. (9pts) Let $f(x) = x^2 - \sqrt{3x - 11} + 3$. Find the following (simplify where appropriate).

$f(9) =$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $f(2) =$

$f(u^2) =$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $f(t + 4) =$

7. (9pts) Find the domains of the functions below and write them using interval notation.

$f(x) = \sqrt{4x - 3}$
$\qquad\qquad\qquad\qquad\qquad\qquad$ $f(x) = \dfrac{27x + 3}{x^2 - 2x - 15}$

8. (6pts) Solve and write the solution in interval notation.

$5 - 2x < 1$ or $9 - 4x > 6$

9. (10pts) A circle centered at $(-3, 1)$ contains the point $(2, -1)$.
a) Find the equation of the circle.
b) Draw the circle in the coordinate plane.

10. (12pts) Cassandra is choosing a cell-phone provider on the basis of data plans, since service already includes unlimited talk and texting. Penny Phone charges $7.50 per month plus $6 per gigabyte of data. Data Boss charges $29 per month, which includes 3 gigabytes of data, plus $4.50 per gigabyte for data above 3 gigabytes. If Cassandra always uses more than 3 gigabytes a month, for which amounts of data is Penny Phone cheaper? Solve as an inequality.

11. (14pts) A steer and rodeo Steve's horse are running in the same direction. At start, the steer is 35 feet away and running at speed 9 feet per second. Steve is following on horse at 14 feet per second.
a) How long until Steve catches up with the steer?
b) How far do Steve and horse go until that moment?

Bonus (10pts) How many milliliters of a 40% solution of isopropyl alcohol must be mixed with 120 milliliters of an 80% solution of isopropyl alcohol in order to get a 65% solution of isopropyl alcohol?

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.
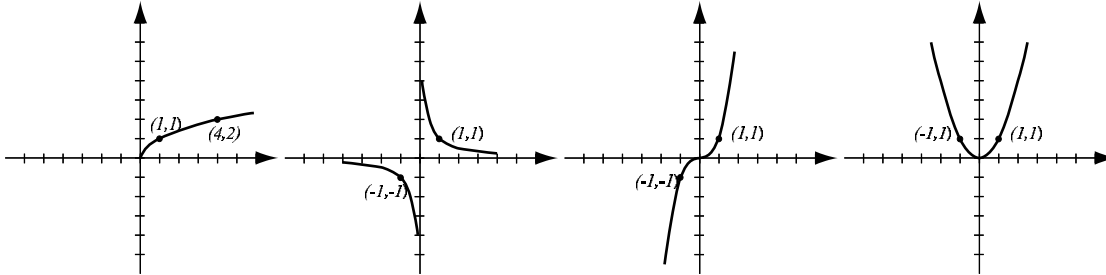
2. (20pts) Let $f(x) = x^2 - x - 12$, $g(x) = \sqrt{2x + 5}$.

Find the following (simplify where possible):

$(f - g)(0) =$ $\dfrac{f}{g}(2) =$

$(fg)(-1) =$ $(g \circ f)(x) =$

$(f \circ g)(x) =$

The domain of $\dfrac{g}{f}$ in interval notation

3. (6pts) Consider the function $h(x) = \sqrt{x^2 + 3}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

4. (6pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = x^3$, stretched horizontally by factor 4.
b) shape of $y = \sqrt{x}$, reflected over the x-axis, then shifted up 2 units.

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(-x) - 2$ and $2f(x - 3)$ and label all the relevant points.
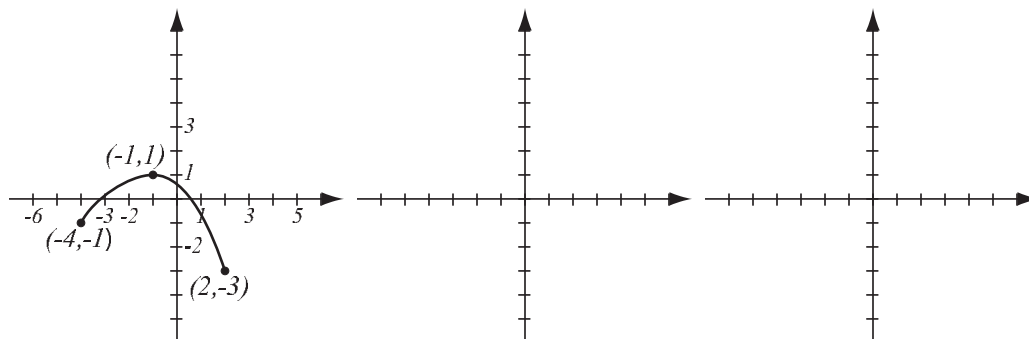
6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} -3x - 5, & \text{if } x \leq -1 \\ x - 2, & \text{if } -1 < x \leq 3 \end{cases}$$

7. (8pts) Find the values of the piecewise-defined function.

$$g(x) = \begin{cases} x^2 - 1, & \text{if } -7 < x \le -2 \\ |x|, & \text{if } -2 < x < 2 \\ \sqrt{3x + 1}, & \text{if } x \ge 2 \end{cases}$$

$g(16) =$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $g(-1) =$

$g(-2) =$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $g(-10) =$

8. (20pts) Let $f(x) = \dfrac{4x}{x^2 + 5}$ (answer with 6 decimal points accuracy).

a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate units on the axes.

b) Determine algebraically whether the function is odd, even, or neither.

c) Verify your conclusion from b) by stating symmetry.

d) Find the local maxima and minima for this function.

e) State the intervals where the function is increasing and where it is decreasing.

9. (14pts) Entrepreneur Edmund is building a block of six self-storage units, with total area 1500 square feet. They are open on one side to accommodate a garage door. Edmund's goal is to minimize building cost, same as minimizing the total length of the walls.

a) Express the total wall length as a function of the length of one of the sides x. What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the block that has the smallest total wall length and what is the smallest total wall length?

y

x
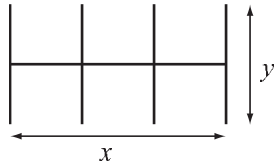
Bonus. (10pts) Let $f(x) = x^2 - 6x - 1$ and $g(x) = 3 - \sqrt{x + 10}$. Find the functions $(f \circ g)(x)$ and $(g \circ f)(x)$.

Simplify, so that the answer is in form $a + bi$.

1. (5pts) $(4 - i)i - 2(5 - 3i) =$

2. (5pts) $\dfrac{3 - 5i}{2 + i} =$

3. (4pts) Simplify and justify your answer.

$i^{403} =$

4. (6pts) Solve the equation by completing the square.

$x^2 - 4x = -29$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|x - 2| < 5$

6. (6pts) Let $P(x)$ be a polynomial of degree 4.
a) What is the maximal number of x-intercepts that $P(x)$ can have? The maximal number of turning points?
b) Draw a graph of P that has exactly 3 x-intercepts and 3 turning points.
c) Draw a graph of P that has exactly 2 turning points, if possible. If not, explain why.

7. (12pts) The quadratic function $f(x) = x^2 - 4x - 21$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

Solve the equations:

8. (8pts) $\dfrac{x+6}{x-4} - \dfrac{4}{x^2-4x} = \dfrac{x+5}{x-4}$

9. (8pts) $2 + \sqrt{22 - x} = x$

10. (14pts) The polynomial $f(x) = (x-3)^2(x+1)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
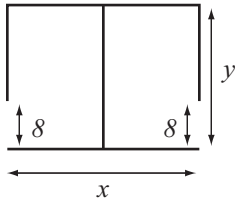c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

11. (12pts) One side of a rectangle is 2 inches longer than the other. If we triple the shorter side and lengthen the longer by 1 inch, we get a rectangle with area 49 square inches greater than the area of the original rectangle. What are the dimensions of the original rectangle?

12. (14pts) A distributor is building a warehouse with two separated areas that have 8-meter openings. The distributor has enough money to build 800 meters of walls, and its goal is to maximize the total area of the warehouse.

a) Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?

Bonus. (10pts) Solve the equation by completing the square. You will need to work with fractions.

$$x^2 + 3x + 1 = 0$$

1. (8pts) Evaluate without using the calculator:

$\log_2 32 =$ $\qquad$ $\log_5 \frac{1}{25} =$ $\qquad$ $\log_a \sqrt[3]{a^8} =$ $\qquad$ $\log_{b^2} b^8 =$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_8 75$ with accuracy 6 decimal places. Show how you obtained your number.

3. (5pts) If $\log_a 4 = u$ and $\log_a 9 = v$, express in terms of u and v:

$\log_a 36 =$ $\qquad\qquad\qquad$ $\log_a \dfrac{81}{16} =$

4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_3 \dfrac{27x^2}{\sqrt{y^5}} =$

5. (6pts) Write as a single logarithm. Simplify if possible.
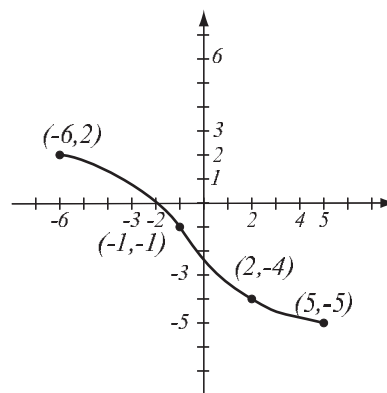
$2\log_7(x^{-2}y^3) - 3\log_7(x^2 y^{-2}) =$

6. (4pts) Simplify.

$\log 10^{5-x} =$ $\qquad\qquad\qquad$ $e^{\ln 5} =$

7. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points, and showing any asymptotes.

8. (9pts) Let $f(x) = \dfrac{x}{2x-5}$.
a) Find the formula for f^{-1}.
b) Find the range of f.

9. (6pts) Using transformations, draw the graph of $f(x) = -\ln x + 3$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.

10. (3pts) Find the domain of the function $f(x) = \log(3 - 7x)$ and write it in interval notation.

11. (9pts) $1800 is deposited in an account bearing 3.25% interest, compounded quarterly. How much is in the account after 4 years?

Solve the equations.

12. (6pts) $9^{4x+1} = 3^{x-7}$

13. (8pts) $4^{x+1} = 7^{x+3}$

14. (8pts) $3^{2x} - 7 \cdot 3^x - 18 = 0$

15. (12pts) Hydrogen-3, a radioactive isotope, decays over time. Starting with 200 grams of hydrogen-3, the amount of it left after t years is given by the function $A(t) = 200 \cdot 0.945^t$.
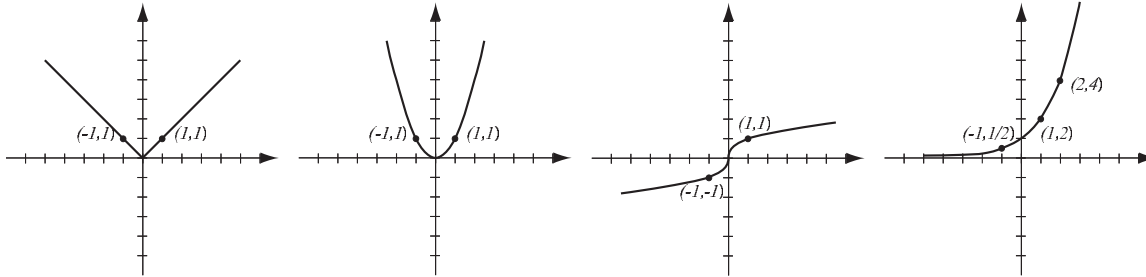a) Graph the amount function.
b) How much hydrogen-3 is left after 6 and 20 years?
c) When will there be 10 grams of hydrogen-3 left?

Bonus (10pts) Let $f(x) = 1 + e^{4x-3}$.
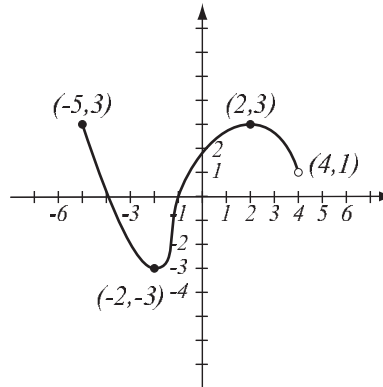a) Find the formula for f^{-1}.
b) Find the range of f.

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.

(-1,1) (1,1)   (-1,1) (1,1)   (1,1) (-1,-1)   (2,4) (-1,1/2) (1,2)

2. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find: $f(-1) =$ $f(4) =$

b) What is the domain of f?

c) What is the range of f?

d) What are the solutions
of the equation $f(x) = 3$?

(-5,3)   (2,3)   (4,1)   (-2,-3)

3. (5pts) Write the equation of the line through points $(-1, 0)$ and $(4, 2)$.

4. (9pts) Find the equation of the line (in form $y = mx + b$) passing through $(2, 5)$ that is parallel to the line $x + 2y = -4$. Draw both lines.
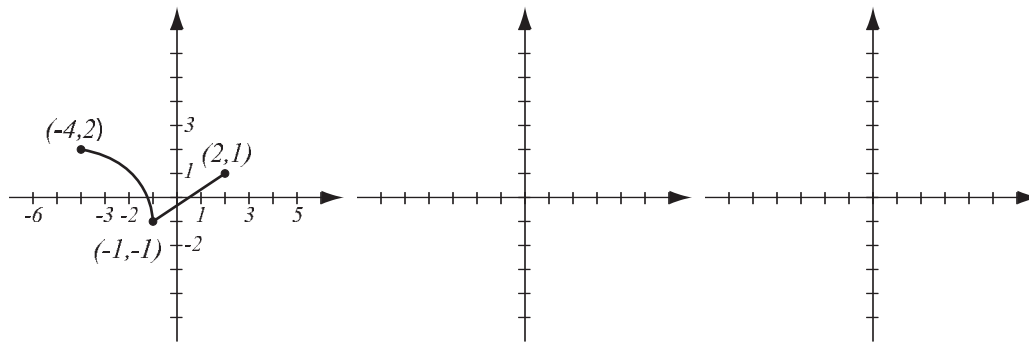
5. (6pts) Solve and write the solution in interval notation.

$|x + 4| < 5$

6. (3pts) Find the domain of the function $f(x) = \sqrt{9 - 4x}$ and write it in interval notation.

7. (4pts) Let $f(x) = 3x + 2$. Find the formula for f^{-1}.

8. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x-1)$ and $2f(x)+1$ and label all the relevant points.

9. (12pts) The quadratic function $f(x) = 4x^2 + 12x - 7$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

10. (7pts) For the function $f(x) = x^4 - 8x^2 - 7$:
a) determine algebraically whether it is odd, even, or neither
b) use the calculator to draw its graph here and verify your conclusion by stating symmetry.

11. (5pts) If $\log_a 3 = u$ and $\log_a 4 = v$, express in terms of u and v:

$\log_a \dfrac{3}{4} =$ $\log_a 48 =$

12. (5pts) Write as a single logarithm. Simplify if possible.

$2\log_2(x^{-3}y^{-2}) + 3\log_2(xy^{-4}) =$

13. (14pts) The polynomial $f(x) = -(x+4)(x-2)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

Solve the equations.

14. (8pts) $\sqrt{x+8} - 6 = x$ **15.** (6pts) $5^{2x+1} = \left(\dfrac{1}{5}\right)^{x+3}$

16. (14pts) A steer and rodeo Steve's horse are running in the same direction. At start, the steer is 22 feet away and running at speed 9 feet per second. Steve is following on horse at 13 feet per second.
a) How long until Steve catches up with the steer?
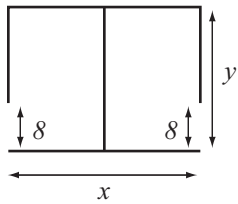b) How far do Steve and horse go until that moment?

17. (12pts) Hydrogen-3, a radioactive isotope, decays over time. Starting with 100 grams of hydrogen-3, the amount of it left after t years is given by the function $A(t) = 100 \cdot 0.945^t$.
a) Graph the amount function.
b) How much hydrogen-3 is left after 4 and 11 years?
c) When will there be 10 grams of hydrogen-3 left?

18. (14pts) A distributor is building a warehouse with two separated areas that have 8-meter openings. The distributor has enough money to build 740 meters of walls, and its goal is to maximize the total area of the warehouse.

a) Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?

Bonus. (10pts) Solve the equation by completing the square. You will need to work with fractions.

$$x^2 + 5x + 2 = 0$$