College Algebra — Final Exam **MAT 140, Fall 2019 — D. Ivanšić**

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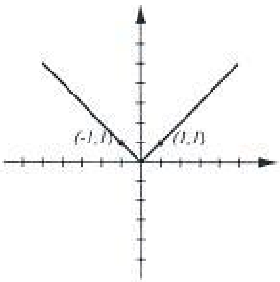
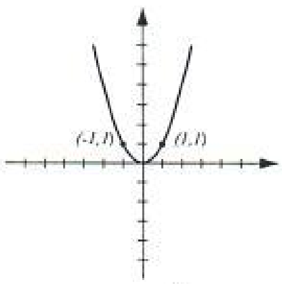
Show all your work!

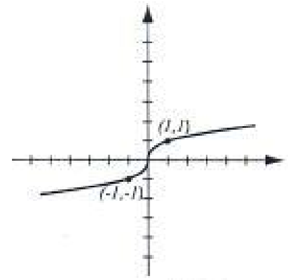
1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



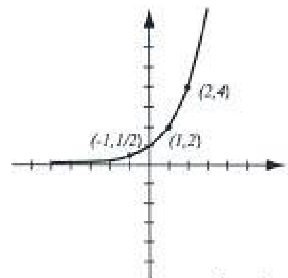
$y = |x|$



$y = x^2$

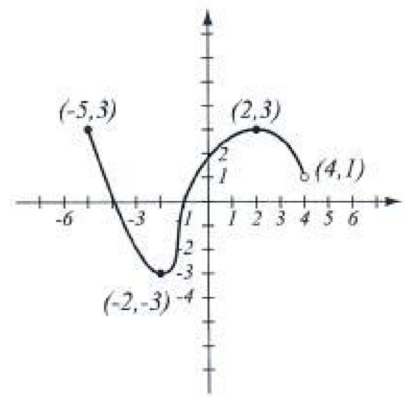


$y = \sqrt[3]{x}$



$y = 2^x$ (or e^x)

2. (8pts) Use the graph of the function f at right to answer the following questions.



a) Find: $f(-1) =$ 0 $\quad f(4) =$ not defined

b) What is the domain of f? $[-5, 4)$

c) What is the range of f? $[-3, 3]$

d) What are the solutions of the equation $f(x) = 3$?

$x = -5, 2$

3. (5pts) Write the equation of the line through points $(-1, 0)$ and $(4, 2)$.

$m = \frac{2-0}{4-(-1)} = \frac{2}{5}$

$y - 0 = \frac{2}{5}(x - (-1))$

$y = \frac{2}{5}x + \frac{2}{5}$

4. (9pts) Find the equation of the line (in form $y = mx + b$) passing through $(2, 5)$ that is parallel to the line $x + 2y = -4$. Draw both lines.

$x + 2y = -4$

$2y = -x - 4 \quad | \div 2$

$y = -\frac{x}{2} - 2$

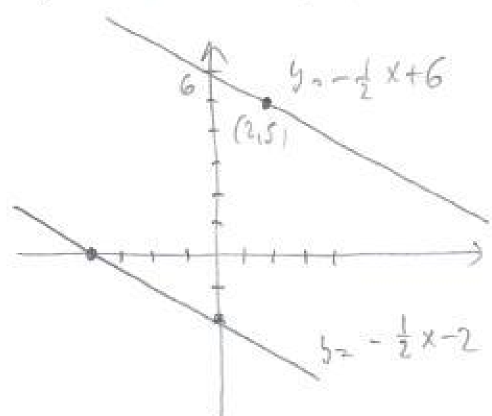
slope $= -\frac{1}{2}$,

slope of parallel line is $-\frac{1}{2}$

$y - 5 = -\frac{1}{2}(x - 2)$

$y = -\frac{1}{2}x + 1 + 5$

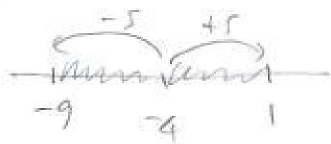
$y = -\frac{1}{2}x + 6$



5. (6pts) Solve and write the solution in interval notation.

$|x+4| < 5$ distance from x to $-4 < 5$

$|x-(-4)| < 5$



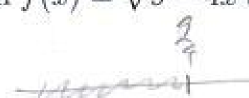
$(-9, 1)$

6. (3pts) Find the domain of the function $f(x) = \sqrt{9-4x}$ and write it in interval notation.

Must have $9-4x \geq 0$

$9 \geq 4x$

$x \leq \frac{9}{4}$



$(-\infty, \frac{9}{4}]$

7. (4pts) Let $f(x) = 3x+2$. Find the formula for f^{-1}.

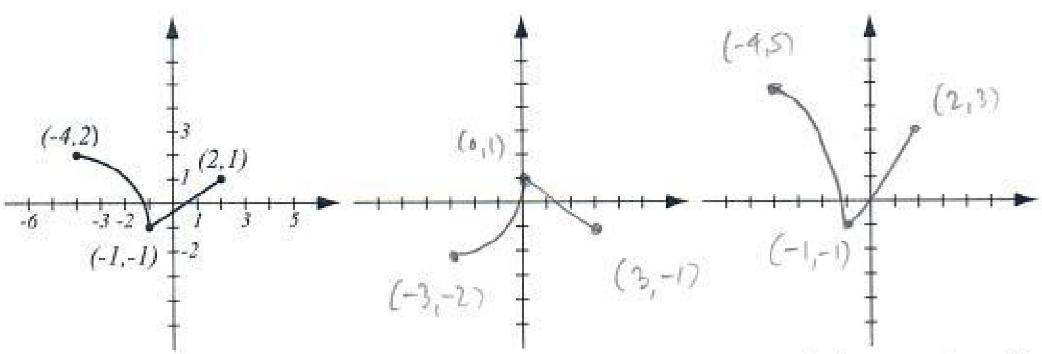
$y = 3x+2$

$y-2 = 3x$

$x = \frac{y-2}{3}$

$f^{-1}(y) = \frac{y-2}{3}$

8. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x-1)$ and $2f(x)+1$ and label all the relevant points.



shift right 1

reflect in x-axis

$x \mapsto x+1$

$y \mapsto -y$

stretch vertically, factor = 2

then, shift up 1

$x \mapsto x$

$y \mapsto 2y+1$

9. (12pts) The quadratic function $f(x) = 4x^2 + 12x - 7$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

a) y-int: $f(0) = -7$

x-int: $4x^2 + 12x - 7 = 0$

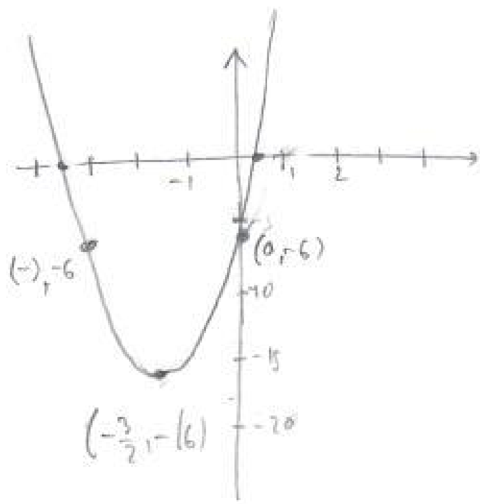
$$x = \frac{-12 \pm \sqrt{12^2 - 4 \cdot 4 \cdot (-7)}}{2 \cdot 4} = \frac{-12 \pm \sqrt{144 + 112}}{8}$$

256

$$= \frac{-12 \pm 16}{8} = -\frac{28}{8}, \frac{4}{8} = -\frac{7}{2}, \frac{1}{2}$$

b) vertex $h = -\frac{b}{2a} = -\frac{12}{2 \cdot 4} = -\frac{3}{2}$

$k = 4\left(-\frac{3}{2}\right)^2 + 12 \cdot \left(-\frac{3}{2}\right) - 7 = 9 - 18 - 7 = -16$

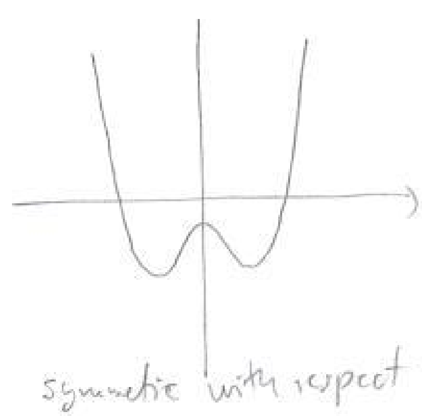


10. (7pts) For the function $f(x) = x^4 - 8x^2 - 7$:
a) determine algebraically whether it is odd, even, or neither
b) use the calculator to draw its graph here and verify your conclusion by stating symmetry.

a) $f(-x) = (-x)^4 - 8(-x)^2 - 7$

$= x^4 - 8x^2 - 7 = f(x)$

so even



symmetric with respect to y-axis

11. (5pts) If $\log_a 3 = u$ and $\log_a 4 = v$, express in terms of u and v:

$\log_a \frac{3}{4} = \log_a 3 - \log_a 4$

$= u - v$

$\log_a 48 = \log_a 3 + \log_a 16$ (3·16)

$= \log_a 3 + \log_a 4^2$

$= \log_a 3 + 2\log_a 4$

$= u + 2v$

12. (5pts) Write as a single logarithm. Simplify if possible.

$2\log_2(x^{-3}y^{-2}) + 3\log_2(xy^{-4}) =$ $\log_2 (x^{-3}y^{-2})^2 + \log_2 (xy^{-4})^2$

$= \log_2 (x^{-6}y^{-4}) + \log_2 (x^2y^{-8}) = \log_2 (x^{-6}y^{-4}x^2y^{-8}) = \log_2 (x^{-4}y^{-12})$

$= \log_2 \frac{1}{x^4y^{12}}$

13. (14pts) The polynomial $f(x) = -(x+4)(x-2)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

a) like $-x \cdot x^2 = -x^3$

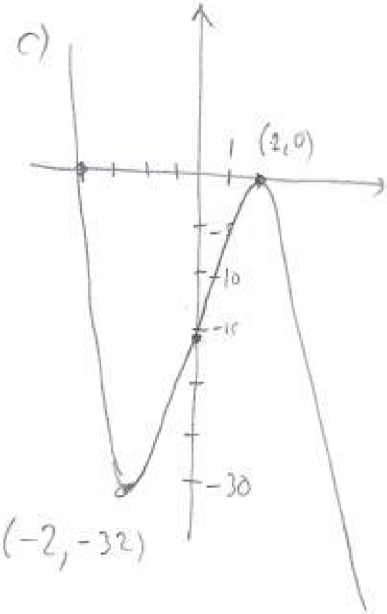
b)

zero	-4	2
mult.	1	2

y-int: $f(0) = -4 \cdot (-2)^2 = -16$

d) Turning points: $(2,0)$
$(-2,-32)$

c)



Solve the equations.

14. (8pts) $\sqrt{x+8} - 6 = x$

$\sqrt{x+8} = x+6 \quad |^2$

$x+8 = x^2 + 2 \cdot x \cdot 6 + 6^2$

$x+8 = x^2 + 12x + 36$

$x^2 + 11x + 28 = 0$

$(x+4)(x+7) = 0$

$x = -4, -7$

only solution

Check:

$\sqrt{-4+8} - 6 = -4$

$2 - 6 = -4$ yes

$\sqrt{-7+8} - 6 = -7$

$1 - 6 = -7$ no

15. (6pts) $5^{2x+1} = \left(\frac{1}{5}\right)^{x+3}$

$5^{2x+1} = (5^{-1})^{x+3}$

$5^{2x+1} = 5^{-x-3}$

$2x+1 = -x-3$

$3x = -4$

$x = -\frac{4}{3}$

16. (14pts) A steer and rodeo Steve's horse are running in the same direction. At start, the steer is 22 feet away and running at speed 9 feet per second. Steve is following on horse at 13 feet per second.

a) How long until Steve catches up with the steer?

b) How far do Steve and horse go until that moment?

steer $d = 9 \cdot t$

$d + 22 = 13 \cdot t$

$9t + 22 = 13t$

$4t = 22$

$t = \frac{22}{4} = \frac{11}{2} = 5.5$

a) Steve catches up after 5.5 seconds

b.) Steve goes $13 \cdot 5.5 = 71.5$ feet

horse goes $9 \cdot 5.5 = 49.5$ feet

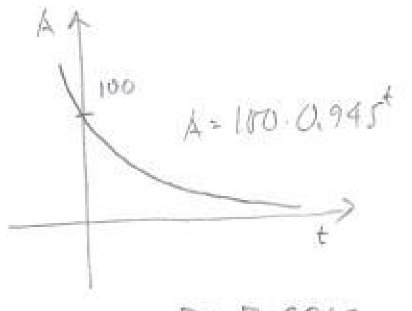
17. (12pts) Hydrogen-3, a radioactive isotope, decays over time. Starting with 100 grams of hydrogen-3, the amount of it left after t years is given by the function $A(t) = 100 \cdot 0.945^t$.

a) Graph the amount function.

b) How much hydrogen-3 is left after 4 and 11 years?

c) When will there be 10 grams of hydrogen-3 left?

a)



b.) $A(4) = 79.749365$ grams

$A(11) = 53.672261$ grams

c) $10 = 100 \cdot 0.945^t \quad | \div 100$

$0.1 = 0.945^t \quad | \ln$

$\ln 0.1 = \ln 0.945^t$

$\ln 0.1 = t \ln 0.945$

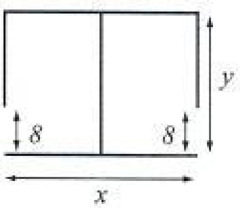
$t = \frac{\ln 0.1}{\ln 0.945} = 40.703037$ years

About 41 years to reach 10 grams.

18. (14pts) A distributor is building a warehouse with two separated areas that have 8-meter openings. The distributor has enough money to build 740 meters of walls, and its goal is to maximize the total area of the warehouse.

a) Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?



a) $2x + y + 2(y-8) = 740$

$2x + 3y - 16 = 740$

$2x + 3y = 756$

$3y = -2x + 756$

$y = -\frac{2}{3}x + 252$

$A = xy = x \cdot \left(-\frac{2}{3}x + 252\right)$

$= -\frac{2}{3}x^2 + 252x$

Domain:

Must have:

$x \geq 0$

$y \geq 8$

$-\frac{2}{3}x + 252 \geq 8$

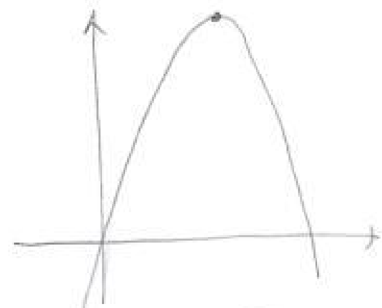
$-2x + 756 \geq 8$

$-2x \geq -748$

$x \leq 374$

Domain $[0, 372]$

b)



$h = -\frac{252}{2\cdot\left(-\frac{2}{3}\right)} = \frac{3\cdot 252}{2\cdot 2} = 189$

$k = 23{,}814 \text{ m}^2$

$y = -\frac{2}{3}\cdot 189 + 252$

Dimensions: 189×126

Biggest area: $23{,}814 \text{ m}^2$

Bonus. (10pts) Solve the equation by completing the square. You will need to work with fractions.

$x^2 + 5x + 2 = 0 \quad \Big| + \left(\frac{5}{2}\right)^2$

$\underbrace{x^2 + 2\cdot x\cdot\frac{5}{2} + \left(\frac{5}{2}\right)^2} + 2 = \left(\frac{5}{2}\right)^2$

$\left(x + \frac{5}{2}\right)^2 = \frac{25}{4} - 2$

$\left(x + \frac{5}{2}\right)^2 = \frac{17}{4}$

$x + \frac{5}{2} = \pm\frac{\sqrt{17}}{2}$

$x = -\frac{5}{2} \pm \frac{\sqrt{17}}{2}$

$x = \frac{-5 \pm \sqrt{17}}{2}$