College Algebra — Exam 4
MAT 140, Fall 2019 — D. Ivanšić

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Show all your work!

1. (8pts) Evaluate without using the calculator:

$\log_2 32 =$ 5 $\quad$ $\log_5 \frac{1}{25} =$ -2 $\quad$ $\log_a \sqrt[3]{a^8} =$ $\frac{8}{3}$ $\quad$ $\log_{b^2} b^8 =$ 4

$2^? = 32$ $\quad$ $5^? = \frac{1}{25} = \frac{1}{5^2} = 5^{-2}$ $\quad$ $a^? = \sqrt[3]{a^8} = a^{\frac{8}{3}}$ $\quad$ $(b^2)^? = b^8 = (b^2)^4$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_8 75$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_8 75 = \frac{\ln 75}{\ln 8} = 2.076273$

3. (5pts) If $\log_a 4 = u$ and $\log_a 9 = v$, express in terms of u and v:

$\log_a 36 = \log_a(4 \cdot 9)$
$= \log_a 4 + \log_a 9$
$= u + v$

$\log_a \frac{81}{16} = \log_a 81 - \log_a 16$
$= \log_a 9^2 - \log_a 4^2$
$= 2\log_a 9 - 2\log_a 4$
$= 2v - 2u$

4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_3 \frac{27x^2}{\sqrt{y^5}} = \log_3 27 + \log_3 x^2 - \log_3 y^{\frac{5}{2}} = 3 + 2\log_3 x - \frac{5}{2}\log_3 y$

$y^{\frac{5}{2}}$ $\quad$ $3^? = 27$

5. (6pts) Write as a single logarithm. Simplify if possible.

$2\log_7(x^{-2}y^3) - 3\log_7(x^2y^{-2}) = \log_7 (x^{-2}y^3)^2 - \log_7 (x^2y^{-2})^3$

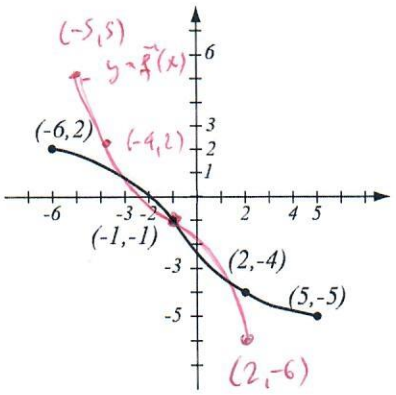
$= \log_7 (x^{-4}y^6) - \log_7 (x^6y^{-6}) = \log_7 \frac{x^{-4}y^6}{x^6y^{-6}}$

$= \log_7 (x^{-10}y^{12}) = \log_7 \frac{y^{12}}{x^{10}}$

6. (4pts) Simplify.

$\log 10^{5-x} =$ $5 - x$ $\quad$ $e^{\ln 5} =$ 5

7. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points, and showing any asymptotes.



a) Yes, passes the horizontal line test

8. (9pts) Let $f(x) = \dfrac{x}{2x-5}$.
a) Find the formula for f^{-1}.
b) Find the range of f.

a) $y = \dfrac{x}{2x-5}$

$(2x-5)y = x$

$2xy - 5y = x$

$2xy - x = 5y$

$x(2y-1) = 5y$

$x = \dfrac{5y}{2y-1} = f^{-1}(y)$

b) range of f = domain of f^{-1}

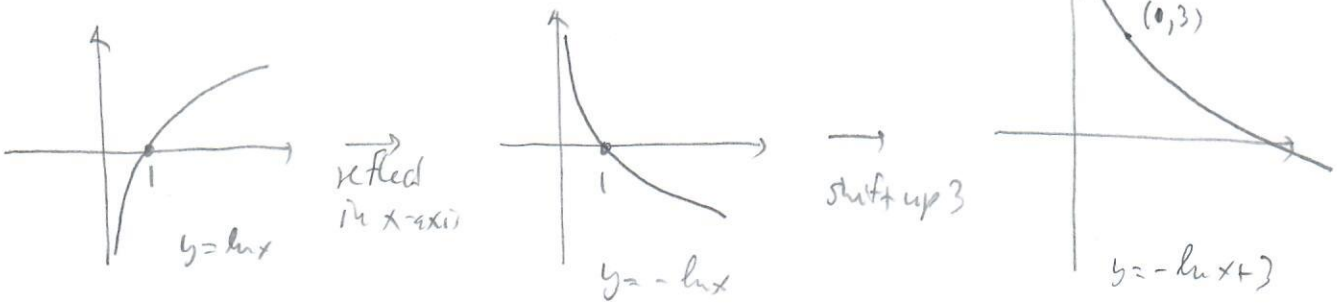
Can't have $2y-1=0$

$y = \frac{1}{2}$

Domain of $f^{-1} = (-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

= range f

9. (6pts) Using transformations, draw the graph of $f(x) = -\ln x + 3$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.



10. (3pts) Find the domain of the function $f(x) = \log(3 - 7x)$ and write it in interval notation.

Must have $3-7x>0$ Domain $(-\infty, \frac{3}{7})$

$3>7x$

$x<\frac{3}{7}$

$\frac{3}{7}$

11. (7pts) \$1800 is deposited in an account bearing 3.25% interest, compounded quarterly. How much is in the account after 4 years?

$A = P\left(1+\frac{r}{n}\right)^{nt} = 1800\left(1+\frac{0.0325}{4}\right)^{4\cdot4} = 1800 \cdot 1.008125^{16}$

$= 1800 \cdot 1.13823\ldots = 2048.81$

Solve the equations.

12. (7pts) $9^{4x+1} = 3^{x-7}$

$(3^2)^{4x+1} = 3^{x-7}$

$3^{8x+2} = 3^{x-7}$

$8x+2 = x-7$

$7x = -9$

$x = -\frac{9}{7}$

13. (8pts) $4^{x+1} = 7^{x+3}$ | ln

$\ln 4^{x+1} = \ln 7^{x+3}$

$(x+1)\ln 4 = (x+3)\ln 7$

$x\ln 4 + \ln 4 = x\ln 7 + 3\ln 7$

$x\ln 4 - x\ln 7 = 3\ln 7 - \ln 4$

$x(\ln 4 - \ln 7) = 3\ln 7 - \ln 4$

$x = \frac{3\ln 7 - \ln 4}{\ln 4 - \ln 7} = -7.954451$

14. (9pts) $3^{2x} - 7\cdot 3^x - 18 = 0$

$(3^x)^2 - 7\cdot 3^x - 18 = 0$

let $u = 3^x$

$u^2 - 7u - 18 = 0$

$(u-9)(u+2) = 0$

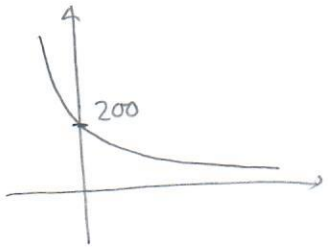
$u = 9, -2$

$3^x = 9$ or $3^x = -2$

$\boxed{x=2}$ not possible

15. (12pts) Hydrogen-3, a radioactive isotope, decays over time. Starting with 200 grams of hydrogen-3, the amount of it left after t years is given by the function $A(t) = 200 \cdot 0.945^t$.
a) Graph the amount function.
b) How much hydrogen-3 is left after 6 and 20 years?
c) When will there be 10 grams of hydrogen-3 left?

a)



b) $A(6) = 200 \cdot 0.945^6 = 142.436354$ grams

$A(20) = 200 \cdot 0.945^{20} = 64.515812$ grams

c) $A(t) = 10$

$200 \cdot 0.945^t = 10$

$0.945^t = \frac{10}{200}$ | ln

$t \ln 0.945 = \ln 0.05$

$t = \frac{\ln 0.05}{\ln 0.945} = 52.955872$

After about 53 years

Bonus (10pts) Let $f(x) = 1 + e^{4x-3}$.
a) Find the formula for f^{-1}.
b) Find the range of f.

a) $y = 1 + e^{4x-3}$

$y - 1 = e^{4x-3}$ | ln

$\ln(y-1) = 4x - 3$

$\ln(y-1) + 3 = 4x$

$x = \frac{\ln(y-1)+3}{4}$

$f^{-1}(y) = \frac{\ln(y-1)+3}{4}$

b) Range f = domain f^{-1}

Must have $y - 1 > 0$

$y > 1$

Range $f = (1, \infty)$