College Algebra — Exam 3 MAT 140, Fall 2019 — D. Ivanšić	**Name:** Saul Ocean *Show all your work!*

Simplify, so that the answer is in form $a + bi$.

1. (5pts) $(4 - i)i - 2(5 - 3i) = 4i - i^2 - 10 + 6i$

$= 10i + 1 - 10 = -9 + 10i$

2. (5pts) $\dfrac{3-5i}{2+i} = \dfrac{3-5i}{2+i}\,\dfrac{2-i}{2-i} = \dfrac{6-10i-3i+5i^2}{2^2-i^2} = \dfrac{6-13i-5}{4-(-1)} = \dfrac{1-13i}{5}$

3. (4pts) Simplify and justify your answer.

$i^{403} = i^{400} \cdot i^3 = \underbrace{(i^4)^{100}}_{=1} \cdot \underbrace{i^2}_{=-1} \cdot i = -i$

4. (6pts) Solve the equation by completing the square.

$x^2 - 4x = -29$ $\quad (+4$

$x^2 - 4x + 4 = -25$

$(x-2)^2 = -25$

$x - 2 = \pm\sqrt{-25}$

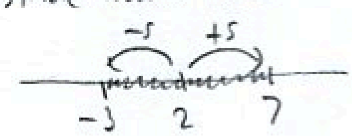
$x - 2 = \pm 5i$

$x = 2 \pm 5i$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|x - 2| < 5$

distance from x to $2 < 5$



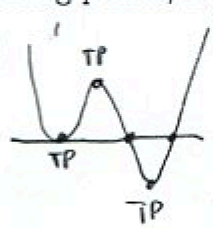
Sol:

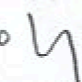
$(-3, 7)$

6. (6pts) Let $P(x)$ be a polynomial of degree 4.
a) What is the maximal number of x-intercepts that $P(x)$ can have? The maximal number of turning points?
b) Draw a graph of P that has exactly 3 x-intercepts and 3 turning points.
c) Draw a graph of P that has exactly 2 turning points, if possible. If not, explain why.

a) max no. of x-int: 4
max no. of turning pts: 3

b)



c) Not possible. To achieve general shape ∪ or ∩ there has to be an odd number of turn. pts.
2 turning points lead to 

or 

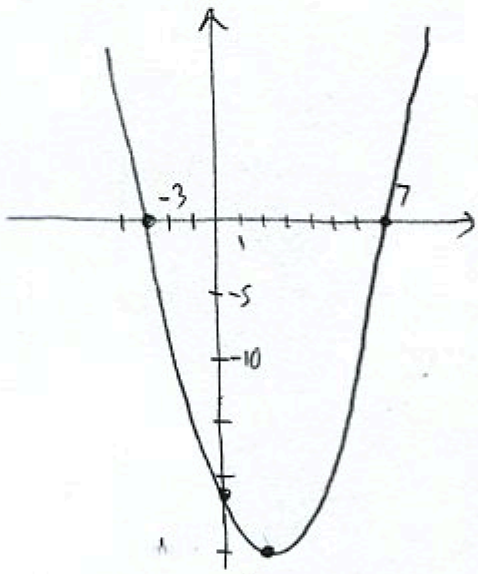
7. (12pts) The quadratic function $f(x) = x^2 - 4x - 21$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

a) y-int: $f(0) = -21$

x-int: $x^2 - 4x - 21 = 0$

$(x-7)(x+3) = 0$

$x = 7, -3$



b) $h = -\frac{b}{2a} = -\frac{-4}{2\cdot 1} = 2$

$k = 2^2 - 4\cdot 2 - 21 = -25$

$(2, -25)$

Solve the equations:

8. (8pts) $\frac{x+6}{x-4} - \frac{4}{x^2-4x} = \frac{x+5}{x-4} \quad \Big|\cdot x(x-4)$

$x(x-4)$

$\frac{x+6}{\cancel{x-4}} \cdot x\cancel{(x-4)} - \frac{4}{\cancel{x(x-4)}}\cancel{x(x-4)} = \frac{x+5}{\cancel{x-4}} \cdot x\cancel{(x-4)}$

$x^2 + 6x - 4 = x^2 + 5x \quad | -x^2 - 5x$

$x - 4 = 0$

$x = 4$

gives 0 in denom., so

$\boxed{\text{no solution}}$

9. (8pts) $2 + \sqrt{22 - x} = x$

$\sqrt{22-x} = x - 2 \quad |^2$

$22 - x = x^2 - 2\cdot x\cdot 2 + 2^2$

$x^2 - 4x + 4 = 22 - x \quad | +x - 22$

$x^2 - 3x - 18 = 0$

$(x+3)(x-6) = 0$

$x = -3, 6$

Check: $2 + \sqrt{22-(-3)} \overset{?}{=} -3$

$2 + \sqrt{25} = -3$ no

$2 + \sqrt{22-6} = 6$

$2 + \sqrt{16} = 6$ yes

$\boxed{x = 6}$ only solution

10. (14pts) The polynomial $f(x) = (x-3)^2(x+1)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

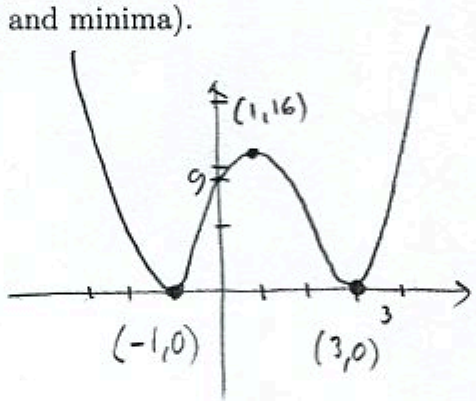
a) Like $x^2 \cdot x^2 = x^4$ $\cup$

b)

zero	3	−1
mult.	2	2

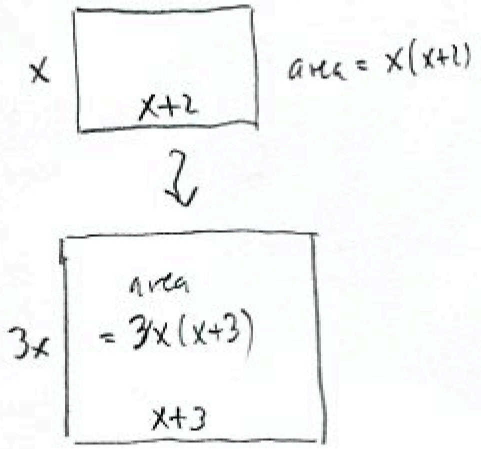
y-int: $f(0) = (-3)^2 \cdot 1^2 = 9$

c)



d) Turning points: $(-1,0)$, $(1,16)$, $(3,0)$

11. (12pts) One side of a rectangle is 2 inches longer than the other. If we triple the shorter side and lengthen the longer by 1 inch, we get a rectangle with area 49 square inches greater than the area of the original rectangle. What are the dimensions of the original rectangle?



$x(x+2) + 49 = 3x(x+3)$

$x^2 + 2x + 49 = 3x^2 + 9x \quad | -x^2 - 2x - 49$

$2x^2 + 7x - 49 = 0$

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 2 \cdot (-49)}}{2 \cdot 2} = \frac{-7 \pm \sqrt{49(1+8)}}{4}$$

$$= \frac{-7 \pm \sqrt{49 \cdot 9}}{4} = \frac{-7 \pm 7 \cdot 3}{4} = \frac{-7 \pm 21}{4} = \frac{7}{2}, -7$$

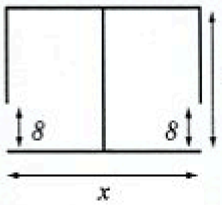
Since $x \geq 0$, only $x = \frac{7}{2}$ is the solution

Rectangle is $\frac{7}{2}$ by $\frac{11}{2}$ (3.5 by 5.5)

12. (14pts) A distributor is building a warehouse with two separated areas that have 8-meter openings. The distributor has enough money to build 800 meters of walls, and its goal is to maximize the total area of the warehouse.

a) Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?



a) $2x + y + 2(y-8) = 800$

$2x + 3y - 16 = 800$

$2x = 816 - 3y$

$x = 408 - \frac{3}{2}y$

$A = xy = \left(408 - \frac{3}{2}y\right)y$

$= -\frac{3}{2}y^2 + 408y$

Domain: Must have

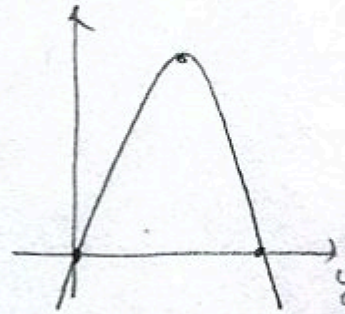
$y \leq \frac{2 \cdot 408}{3}$ $\quad y \geq 8$

$x \geq 0$

$y \leq 272$ $\quad 408 - \frac{3}{2}y \geq 0$

Domain $[0, 272)$ $\quad \frac{3}{2}y \leq 408$

b)



$h = -\frac{b}{2a} = -\frac{408}{2\cdot\left(-\frac{3}{2}\right)} = \frac{408}{3} = 136$

$k = -\frac{3}{2}136^2 + 408 \cdot 136 = 27744$

Dimensions 204×136

Max area: $27{,}744$ m^2

Bonus. (10pts) Solve the equation by completing the square. You will need to work with fractions.

$x^2 + 3x + 1 = 0$ $\quad + \left(\frac{3}{2}\right)^2$

$\underbrace{x^2 + 2 \cdot x \cdot \frac{3}{2} + \left(\frac{3}{2}\right)^2} + 1 = \left(\frac{3}{2}\right)^2$

$\left(x + \frac{3}{2}\right)^2 = \frac{9}{4} - 1$

$\left(x + \frac{3}{2}\right)^2 = \frac{5}{4}$

$x + \frac{3}{2} = \pm\sqrt{\frac{5}{4}}$

$x + \frac{3}{2} = \pm\frac{\sqrt{5}}{2}$

$x = -\frac{3}{2} \pm \frac{\sqrt{5}}{2} = \frac{-3 \pm \sqrt{5}}{2}$