College Algebra — Exam 2
MAT 140, Fall 2019 — D. Ivanšić

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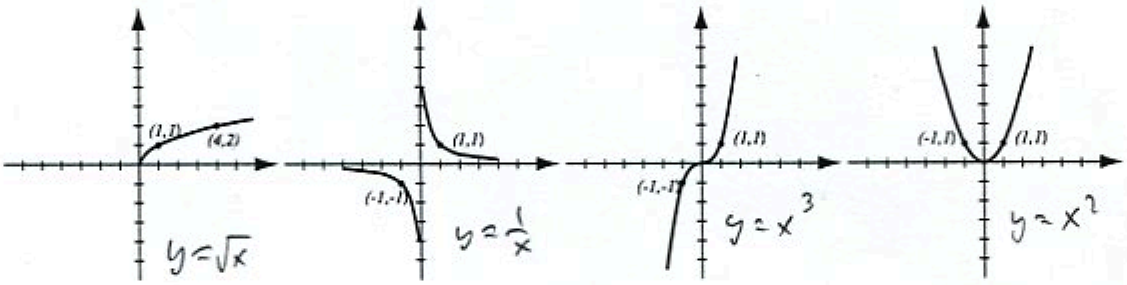
Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (20pts) Let $f(x) = x^2 -$ [redacted]$x -$ [redacted] 12, $g(x) = \sqrt{2x+5}$.

Find the following (simplify where possible):

$(f-g)(0) = f(0)-g(0)$
$= -12-\sqrt{5}$

$\frac{f}{g}(2) = \frac{f(2)}{g(2)} = \frac{4-2-12}{\sqrt{4+5}} = -\frac{10}{3}$

$(fg)(-1) = f(-1)\cdot g(-1)$
$=(1-(-1)-12\cdot\sqrt{-2+5} = -10\sqrt{3}$

$(g\circ f)(x) = g(f(x)) = g(x^2-x-12)$
$= \sqrt{2(x^2-x-12)+5} = \sqrt{2x^2-2x-19}$

$(f\circ g)(x) = f(g(x)) = f(\sqrt{2x+5}) = (\sqrt{2x+5})^2 - \sqrt{2x+5}-12$
$= 2x+5-\sqrt{2x+5}-12 = 2x-7-\sqrt{2x+5}$

The domain of $\frac{g}{f}$ in interval notation D

Domain f: all real numbers
Domain g: must have $2x+5\geq 0$
$2x\geq -5$
$x\geq -\frac{5}{2}$

Can't have $f(x)=0$
$x^2-x-12=0$
$(x-4)(x+3)=0$
$x=4,-3$

~~f~~
~~g~~
~~ans~~
-3 $-\frac{5}{2}$ 4

$[-\frac{5}{2},4)\cup(4,\infty)$

3. (6pts) Consider the function $h(x) = \sqrt{x^2+3}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

$g(x) = x^2+3$ $\qquad\qquad$ $g(x) = x^2$

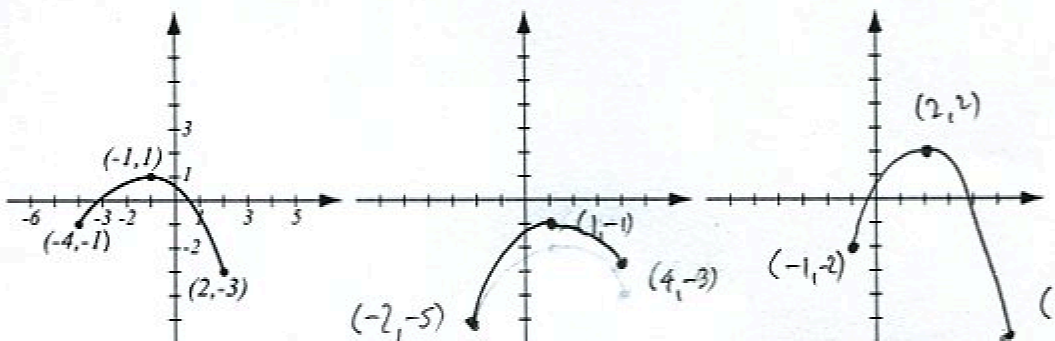
$f(x) = \sqrt{x}$ $\qquad\qquad$ $f(x) = \sqrt{x+3}$

4. (6pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = x^3$, stretched horizontally by factor 4.
b) shape of $y = \sqrt{x}$, reflected over the x-axis, then shifted up 2 units.

a) $x^3 \xrightarrow[\text{x by } \frac{1}{4}x]{\text{replace}} \left(\frac{1}{4}x\right)^3$

b) $\sqrt{x} \xrightarrow[\text{by } -1]{\text{mult}} -\sqrt{x} \xrightarrow[\text{add } 2]{} -\sqrt{x}+2$

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(-x)-2$ and $2f(x-3)$ and label all the relevant points.



reflect in y-axis
shif down 2

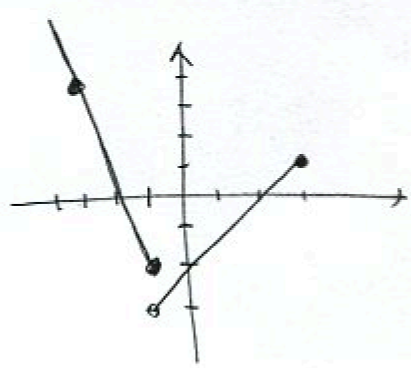
shift right 3
stetch vertically, Factor = 2

6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} -3x-5, & \text{if } x \leq -1 \\ x-2, & \text{if } -1 < x \leq 3 \end{cases}$$

x	$-3x-5$
-1	-2
-3	4

x	$x-2$
-1	-3
3	1



7. (8pts) Find the values of the piecewise-defined function.

$$g(x) = \begin{cases} x^2 - 1, & \text{if } -7 < x \leq -2 \\ |x|, & \text{if } -2 < x < 2 \\ \sqrt{3x+1}, & \text{if } x \geq 2 \end{cases}$$

$g(16) =$ $\sqrt{3 \cdot 16 + 1} = \sqrt{49} = 7$

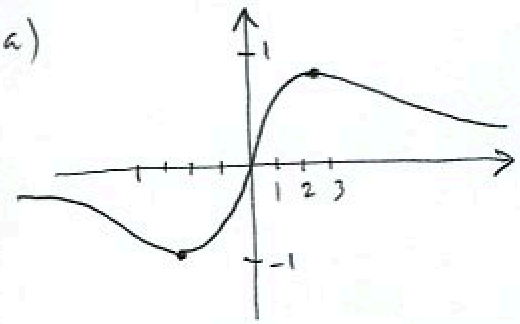
$g(-1) =$ $|-1| = 1$

$g(-2) =$ $(-2)^2 - 1 = 3$

$g(-10) =$ not defined (not in domain

8. (20pts) Let $f(x) = \dfrac{4x}{x^2+5}$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate units on the axes.
b) Determine algebraically whether the function is odd, even, or neither.
c) Verify your conclusion from b) by stating symmetry.
d) Find the local maxima and minima for this function.
e) State the intervals where the function is increasing and where it is decreasing.

a)



b) $f(-x) = \dfrac{4(-x)}{(-x)^2+5} = \dfrac{-4x}{x^2+5} = -f(x)$ odd

c) Graph is symmetric wrt origin

d) $f(2.236068) = 0.894427$ is a local max

$f(-2.236068) = -0.894427$ is a local min

(by symmetry)

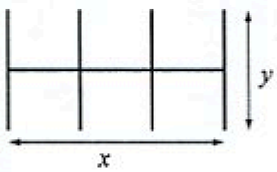
e) increasing on

$(-2.236068, 2.236068)$

decreasing on

$(-\infty, -2.236068)$ and $(2.236068, \infty)$

9. (14pts) Entrepreneur Edmund is building a block of six self-storage units, with total area 1500 square feet. They are open on one side to accommodate a garage door. Edmund's goal is to minimize building cost, same as minimizing the total length of the walls.
a) Express the total wall length as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the block that has the smallest total wall length and what is the smallest total wall length?



a) $l = x + 4y = x + \frac{4 \cdot 1500}{x} = x + \frac{6000}{x}$

$x \cdot y = 1500$

$y = \frac{1500}{x}$

Domain: $x > 0$

$y > 0$

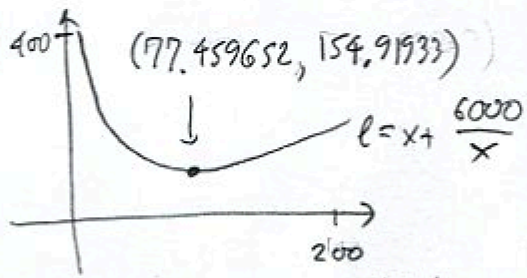
$\frac{1500}{x} > 0$

$1500 > 0$

true, so no restriction

$(0, \infty)$

b)



$1500/x$

Dimensions are $77.459652 \times 19.364921$

Minimal wall length 154.91933 ft

Bonus. (10pts) Let $f(x) = x^2 - 6x - 1$ and $g(x) = 3 - \sqrt{x+10}$. Find the functions $(f \circ g)(x)$ and $(g \circ f)(x)$.

$$f(g(x)) = f(3 - \sqrt{x+10}) = (3 - \sqrt{x+10})^2 - 6(3 - \sqrt{x+10}) - 1$$

$$= 9 - 2 \cdot 3\cancel{\sqrt{x+10}} + \sqrt{x+10}^2 - 18 + 6\cancel{\sqrt{x+10}} - 1$$

$$= 9 + x + 10 - 18 - 1 = x$$

$$g(f(x)) = g(x^2 - 6x - 1) = 3 - \sqrt{x^2 - 6x - 1 + 10} = 3 - \sqrt{x^2 - 6x + 9} = 3 - \sqrt{(x-3)^2}$$

$$= 3 - (x - 3) = 6 - x$$