

1. (8pts) Use the graph of the function f at right to answer the following questions.

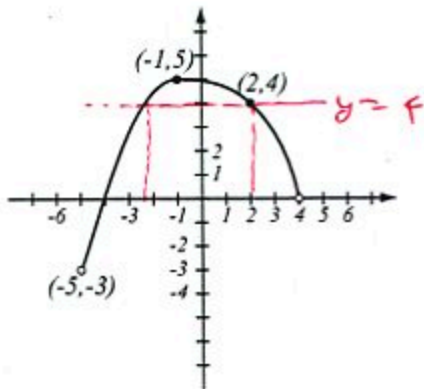
a) Find: $f(-1) = 5$ $f(6) =$ *not defined*

b) What is the domain of f ? $(-5, 4)$

c) What is the range of f ? $(-3, 5]$

d) What are the solutions of the equation $f(x) = 4$?

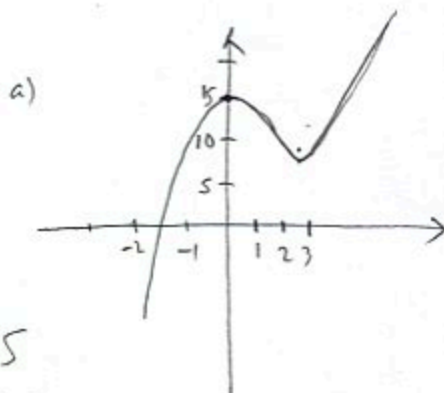
$x = 2, -2.3$



2. (10pts) Use your calculator to accurately sketch the graph of $y = x^3 - 4x^2 + x + 15$.

a) Draw the graph on paper and indicate units on the axes.

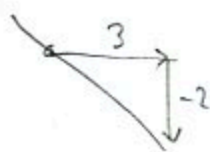
b) Find all the x - and y -intercepts (accuracy: 6 decimal points).



y -int: $f(0) = 15$

x -int: -1.555622

3. (5pts) A line contains the point $(1, -2)$. If you start at any point on the line, go right 3 units and then down 2 units, you wind up back on the line. Write the equation of the line.



slope = $-\frac{2}{3}$

$y - (-2) = -\frac{2}{3}(x - 1)$

$y + 2 = -\frac{2}{3}x + \frac{2}{3}$

$y = -\frac{2}{3}x - \frac{4}{3}$

4. (10pts) Find the equation of the line (in form $y = mx + b$) that is perpendicular to the line $3x - y = 2$ and passes through $(6, -3)$. Draw both lines.

$3x - y = 2$

$-y = -3x + 2$

$y = 3x - 2$

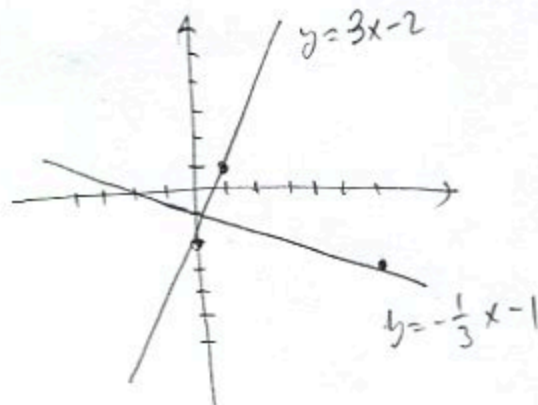
slope of perp. line

$= -\frac{1}{3}$

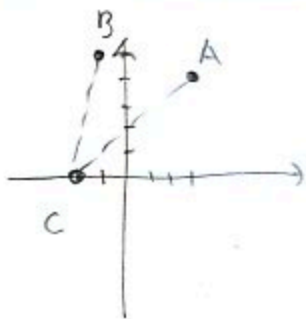
$y - (-3) = -\frac{1}{3}(x - 6)$

$y + 3 = -\frac{1}{3}x + 2$

$y = -\frac{1}{3}x - 1$



5. (7pts) Draw the points $A = (3, 4)$, $B = (-1, 5)$ and $C = (-2, 0)$ in the coordinate plane. Use the distance formula to determine which of A and B is closer to C .



$$d(A, C) = \sqrt{(-2-3)^2 + (0-4)^2} = \sqrt{25+16} = \sqrt{41}$$

$$d(B, C) = \sqrt{(-2-(-1))^2 + (0-5)^2} = \sqrt{1+25} = \sqrt{26}$$

B is closer to C .

6. (9pts) Let $f(x) = x^2 - \sqrt{3x-11} + 3$. Find the following (simplify where appropriate).

$$\begin{aligned} f(9) &= 9^2 - \sqrt{3 \cdot 9 - 11} + 3 \\ &= 81 - \sqrt{16} + 3 = 80 \end{aligned}$$

$$\begin{aligned} f(u^2) &= (u^2)^2 - \sqrt{3u^2 - 11} + 3 \\ &= u^4 - \sqrt{3u^2 - 11} + 3 \end{aligned}$$

$$\begin{aligned} f(2) &= 2^2 - \sqrt{3 \cdot 2 - 11} + 3 = 4 - \sqrt{-5} + 3 \\ &\text{not defined} \end{aligned}$$

$$\begin{aligned} f(t+4) &= (t+4)^2 - \sqrt{3(t+4) - 11} + 3 \\ &= t^2 + 2 \cdot t \cdot 4 + 4^2 - \sqrt{3t + 12 - 11} + 3 \\ &= t^2 + 8t + 19 - \sqrt{3t+1} \end{aligned}$$

7. (9pts) Find the domains of the functions below and write them using interval notation.

$$f(x) = \sqrt{4x-3}$$

Must have

$$4x-3 \geq 0$$

$$4x \geq 3$$

$$x \geq \frac{3}{4}$$

$$\left[\frac{3}{4}, \infty \right)$$

$$f(x) = \frac{27x+3}{x^2-2x-15}$$

Can't have $x^2-2x-15=0$

$$(x-5)(x+3) \neq 0$$

$$x \neq 5, -3$$

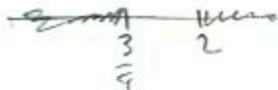
$$(-\infty, -3) \cup (-3, 5) \cup (5, \infty)$$

8. (6pts) Solve and write the solution in interval notation.

$$5 - 2x < 1 \text{ or } 9 - 4x > 6$$

$$-2x < -4 \quad -4x > -3$$

$$x > 2 \text{ or } x < \frac{3}{4}$$



$$(-\infty, \frac{3}{4}) \cup (2, \infty)$$

9. (10pts) A circle centered at $(-3, 1)$ contains the point $(2, -1)$.

a) Find the equation of the circle.

b) Draw the circle in the coordinate plane.

d) $r =$ distance from $(-3, 1)$ to $(2, -1)$

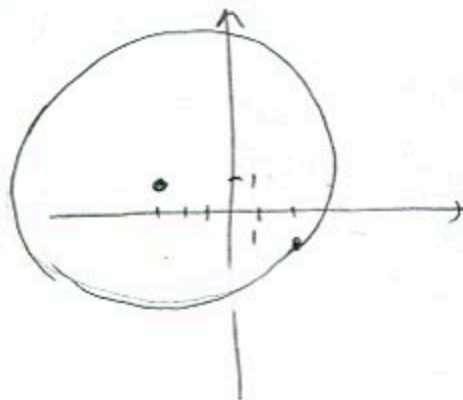
$$= \sqrt{(2 - (-3))^2 + (-1 - 1)^2}$$

$$= \sqrt{25 + 4} = \sqrt{29}$$

$$(x - (-3))^2 + (y - 1)^2 = \sqrt{29}^2$$

$$(x + 3)^2 + (y - 1)^2 = 29$$

eq. of circle



10. (12pts) Cassandra is choosing a cell-phone provider on the basis of data plans, since service already includes unlimited talk and texting. Penny Phone charges \$7.50 per month plus \$6 per gigabyte of data. Data Boss charges \$29 per month, which includes 3 gigabytes of data, plus \$4.50 per gigabyte for data above 3 gigabytes. If Cassandra always uses more than 3 gigabytes a month, for which amounts of data is Penny Phone cheaper? Solve as an inequality.

$x =$ gigabytes used

Penny Phone cost: $7.50 + 6x$

Data Boss cost: $29 + 4.50(x - 3)$

Wish to have $7.50 + 6x \leq 29 + 4.5(x - 3)$

$$7.50 + 6x \leq 29 + 4.5x - 13.50 \quad | -4.5x - 7.5$$

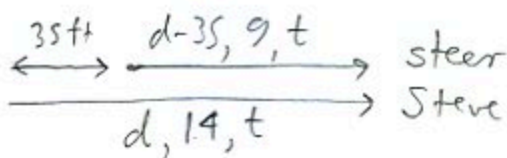
$$1.5x \leq 8$$

$$x \leq \frac{8}{1.5}$$

$$x \leq 5.333333 \text{ gigabytes}$$

11. (14pts) A steer and rodeo Steve's horse are running in the same direction. At start, the steer is 35 feet away and running at speed 9 feet per second. Steve is following on horse at 14 feet per second.

- a) How long until Steve catches up with the steer?
 b) How far do Steve and horse go until that moment?



$$\begin{cases} d-35 = 9t \\ d = 14t \end{cases}$$

a) It takes 7 seconds

$$d = 14 \cdot 7 = 98$$

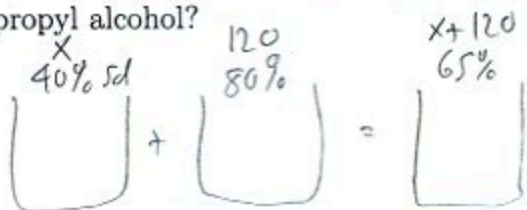
Steve travels 98 ft

$$14t - 35 = 9t \quad | -9t + 35$$

$$5t = 35$$

$$t = 7$$

Bonus (10pts) How many milliliters of a 40% solution of isopropyl alcohol must be mixed with 120 milliliters of an 80% solution of isopropyl alcohol in order to get a 65% solution of isopropyl alcohol?



x = amount of 40% sol. added

$$0.4x + 0.8 \cdot 120 = 0.65(x+120)$$

$$0.4x + 96 = 0.65x + 78 \quad \left| \begin{array}{l} -0.4x \\ -78 \end{array} \right.$$

$$18 = 0.25x$$

$$x = \frac{18}{0.25} = 72 \text{ milliliters}$$

← amount of pure isopropyl alcohol in each container