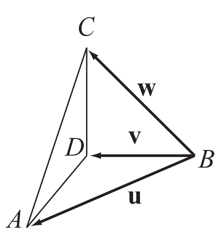
Calculus 3 — Exam 1
MAT 309, Spring 2018 — D. Ivanšić

Name: ____________________

Show all your work!

1. (11pts) Let $\mathbf{u} = \langle 3, -7, 1\rangle$ and $\mathbf{v} = \langle 3, 0, -4\rangle$.
a) Calculate $3\mathbf{u}$, $2\mathbf{u} - \mathbf{v}$, and $\mathbf{u} \cdot \mathbf{v}$.
b) Find a vector of length 4 in direction of $\mathbf{u}$.
c) Find the projection of $\mathbf{u}$ onto $\mathbf{v}$.

2. (12pts) In the picture, a tetrahedron $ABCD$ is given. All edges that meet at D are perpendicular to each other and have length 4. Note that this makes ABC an equilateral triangle. Draw the vectors $\mathbf{u} \times \mathbf{v}$ and $\mathbf{w} \times \mathbf{u}$ and determine their length.



3. (8pts) Draw the region in $\mathbf{R}^3$ described by:

$x^2 + z^2 \leq 4,\ -2 \leq y \leq 5$

4. (12pts) Find the equation of the plane that contains the points $A = (0, 3, -4)$, $B = (3, 1, -2)$ and $C = (4, 0, 2)$.

5. (16pts) This problem is about the surface $\dfrac{x^2}{4} - \dfrac{y^2}{9} - \dfrac{z^2}{1} = 1$.

a) Identify and sketch the intersections of this surface with the coordinate planes.

b) Sketch the surface in 3D, with coordinate system visible.

6. (14pts) The curve $\mathbf{r}(t) = \langle t, t^2 + 1, \sin t \rangle$ is given, t any real number.
a) Sketch the curve in the coordinate system.
b) Find parametric equations of the tangent line to this curve when $t = 0$ and sketch the tangent line.

7. (13pts) Find the length of the curve $\mathbf{r}(t) = \left\langle t\cos t, t\sin t, \frac{2\sqrt{2}}{3}t^{\frac{3}{2}} \right\rangle$, $0 \leq t \leq 4$.

8. (14pts) Suppose the corner of a room is represented by the three coordinate planes. An egg is launched from point $(3, 7, 6)$ with initial velocity vector $\mathbf{v_0} = \langle -5, -2, 7 \rangle$.
a) Assuming gravity acts in the usual negative z-direction (let $g = 10$), find the vector function $\mathbf{r}(t)$ representing the egg's position.
b) On which wall or floor does the egg splatter?

Bonus (10pts) For every pair of skew (nonintersecting) lines in $\mathbf{R}^3$, there is a line that intersects them both and is perpendicular to both. Find this line for the two lines given parametrically below. *Hint: if P is a point on one of the lines, and Q on the other, what conditions have to be satisfied so that the line determined by P and Q is the requested line?*

$$\begin{array}{ll} x = -8 + 5t & x = 3 + 2s \\ y = -1 + t & y = -4 - 2s \\ z = -5 + 2t & z = 2 - s \end{array}$$

Calculus 3 — Exam 2
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Name: ______________________________

Show all your work!

1. (10pts) Let $h(x, y) = x^2 + 4y^2$.
a) Find the domain of h.
b) Sketch the contour map for the function, drawing level curves for levels $k = -1, 0, 1, 4$. Note the domain on the picture.
c) Without computation, draw the directions of $\nabla h(1, 0)$ and $\nabla h(\sqrt{2}, \frac{1}{\sqrt{2}})$. Note that these points are on the level curves you drew in b).

2. (16pts) Let $f(x, y) = \dfrac{\sin^2 x}{\cos^2 y}$.
a) At point $(\frac{\pi}{4}, \frac{\pi}{3})$, find the directional derivative of f in the direction of $\langle 1, 3 \rangle$.
b) In what direction is the directional derivative the greatest, and what is its value?

3. (12pts) Find the equation of the tangent plane to the hyperbolic paraboloid $y = 2x^2 - 3z^2$ at the point $(-1, -10, 2)$. Simplify the equation to standard form.

4. (18pts) Let $W = \dfrac{x}{y-x}$, $x = te^{st}$, $y = s^2 + t^2$. Use the chain rule to find $\dfrac{\partial W}{\partial t}$ when $s = 0$, $t = 2$.

5. (12pts) The body mass index or BMI is calculated using the formula $BMI = \frac{w}{h^2}$, where w and h are weight and height of an individual in kilograms and meters, respectively. Use differentials to estimate the change in BMI if a 1-meter high child weighing 15 kg grows by 1.5cm in height and 0.5kg in weight.

6. (12pts) Use implicit differentiation to find $\frac{\partial z}{\partial y}$ at the point $(1, 1, 1)$, if $\sqrt{x} - \sqrt{y} + \sqrt{z} + \ln(xyz) = 1$.

7. (20pts) Find and classify the local extremes for $f(x, y) = x^2 - xy^2 + y^2$.

Bonus (10pts) Among all rectangular boxes of volume 1, find the one with the shortest diagonal d. *Hint: minimize* d^2.

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Show all your work!

1. (17pts) Let D be the region in the first quadrant bounded by the curves $y = x^2 + 1$, $x = 0$ and $y = 5$.
a) Sketch the region D.
b) Set up $\displaystyle\iint_D \frac{1}{\sqrt{y-1}}\,dA$ as iterated integrals in both orders of integration.
c) Evaluate the double integral using the easier order.

2. (17pts) Find $\iint_D xy\,dA$ if D is the triangle bounded by $y = 1 - x$, $y = x - 3$ and $y = 3$. Sketch the region of integration first.

3. (20pts) Use polar coordinates to find $\iint_D \frac{y}{\sqrt{x^2+y^2}}\,dA$, if D is the region inside the circle $(x-1)^2+y^2=1$, outside the circle $x^2+y^2=2$ and above the x-axis. Sketch the region of integration first.

4. (18pts) Sketch the region E in the first octant $(x, y, z \geq 0)$ that is inside the sphere $x^2+y^2+z^2=1$ and above the plane $z=2y$. Then write the two iterated triple integrals that stand for $\iiint_E f\,dV$ which end in $dx\,dz\,dy$ and $dz\,dy\,dx$.

5. (14pts) Use spherical coordinates to set up the triple integral for the volume of the region that is between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 25$, above the xy-plane, and between the planes $y = \sqrt{3}x$ and $y = -\sqrt{3}x$, the part where $y \geq 0$. Do not evaluate the integral. Sketch the region E.

6. (14pts) Use cylindrical coordinates to set up $\displaystyle\iiint_E \frac{x^2 + y^2 + z^2}{x^2 + y^2 + 1}\, dV$, where E is the region bounded by the paraboloids $z = x^2 + y^2 - 3$ and $z = 9 - x^2 - y^2$. Do not evaluate the integral. Sketch the region E.

Bonus (10pts) Do problem 4 for the iterated triple integral that ends in $dy\,dz\,dx$.

Calculus 3 — Exam 4
MAT 309, Spring 2018 — D. Ivanšić

Name: ____________________

Show all your work!

1. (12pts) Let $\mathbf{F}(x,y) = \langle x^2 - y^2, xy \rangle$.
a) Sketch the vector field by evaluating it at 9 points (for example, a 3×3 grid).
b) Is F conservative?

2. (20pts) In both cases set up and simplify the set-up, but do NOT evaluate the integral.
a) $\int_C \frac{xy}{x^2+y^2}\,ds$, where C is the part of the parabola $y = \sqrt{x}$ from $(1,1)$ to $(4,2)$.
b) $\int_C \mathbf{F} \cdot d\mathbf{r}$, if $\mathbf{F}(x,y,z) = \left\langle -z, -z, \frac{x-y}{x^2+y^2+z^2} \right\rangle$, where C is the curve
$x = \cos t$, $y = -\cos t$, $z = \sqrt{2}\sin t$, $0 \le t \le 2\pi$.

3. (12pts) Let $\mathbf{F}(x, y) = \langle 6x + 7y, 7x - 10y \rangle$. It is easy to see that $\mathbf{F} = \nabla f$, where $f(x, y) = 3x^2 + 7xy - 5y^2$. Apply the fundamental theorem for line integrals to:

a) Find $\int_C \mathbf{F} \cdot d\mathbf{r}$, if C is the unit circle.

b) Find $\int_C \mathbf{F} \cdot d\mathbf{r}$, if C is a curve from $(0, 0)$ to $(3, 1)$. (Why is the curve not specified?)

4. (20pts) Consider the region inside the circle $x^2 + y^2 = 4$ and above the line $y = x$.

a) Draw the region.

b) Use Green's theorem to find the line integral $\int_C y^2\, dx + \frac{x^2}{2}\, dy$, where C is the boundary of the region D, traversed counterclockwise.

5. (22pts) Let D be the region enclosed by the hyperbola $y^2 - x^2 = 1$ and the line $y = 2$. Draw the region.
a) Write the double integral for the area of D.
b) Use Green's theorem to write the integrals that give the area of D.
In both a) and b), simplify until you encounter a hard integral.

6. (14pts) Let $\mathbf{F}(x, y, z) = \langle z^2 - y, \cos z - x, 2xz - y \sin z \rangle$.
a) Find the curl of $\mathbf{F}$.
b) Is $\mathbf{F}$ is conservative? If so, find its potential function.

Bonus. (10pts) Show that the expressions you got in a) and b) of problem 5 are the same number. (You don't have to evaluate the integrals to see this — manipulate one to get the other.)

Calculus 3 — Final Exam
MAT 309, Spring 2018 — D. Ivanšić

Name: ______________________________

Show all your work!

1. (12pts) Find the equation of the plane that contains the point $A = (3, 1, -2)$ and the line given by parametric equations $x = 3 - 2t$, $y = 4$, $z = -1 + t$.

2. (20pts) Let $f(x, y) = \dfrac{1}{x^2 + y^2}$.

a) Sketch the contour map for the function, drawing level curves for levels $k = 0, \frac{1}{4}, 1, 4$.

b) At point $(1, -2)$, find the directional derivative of f in the direction of $\langle -2, -3 \rangle$. In what direction is the directional derivative the greatest? What is the directional derivative in that direction?

c) Let $\mathbf{F} = \nabla f$. Sketch the vector field $\mathbf{F}$.

Apply the fundamental theorem for line integrals to answer:

d) What is $\int_C \mathbf{F} \cdot d\mathbf{r}$, if C is the straight line from $(1, 1)$ to $(3, 0)$?

e) What is $\int_C \mathbf{F} \cdot d\mathbf{r}$, if C is a curve going from any point on level curve $k = 4$ to any point on level curve $k = 1$?

3. (14pts) The curve $\mathbf{r}(t) = \langle t+3, \cos t, \sin t \rangle$ is given, t any real number.
a) Sketch the curve in the coordinate system.
b) Find parametric equations of the tangent line to this curve when $t = 0$ and sketch the tangent line.

4. (14pts) Find and classify the local extremes for $f(x, y) = 2x^2 - 5y^2 - 2xy + 6x - 14y$.

5. (16pts) Let D be the region in the first quadrant bounded by the curves $y = x^2$, $y = 0$ and $x = 2$.

a) Sketch the region D.

b) Set up $\displaystyle\iint_D e^{x^3}\, dA$ as iterated integrals in both orders of integration.

c) Evaluate the double integral using the easier order.

6. (18pts) Sketch the region E in the first octant $(x, y, z \geq 0)$ that is inside the sphere $x^2 + y^2 + z^2 = 1$ and above the plane $z = 2y$. Then write the two iterated triple integrals that stand for $\iiint_E f\, dV$ which end in $dx\, dz\, dy$ and $dz\, dy\, dx$.

7. (14pts) Use either cylindrical or spherical coordinates to set up $\iiint_E \frac{x^2+y^2+z^2}{x^2+1}\, dV$, where E is the region inside the sphere $x^2 + y^2 + z^2 = 4$ and above the plane $z = \sqrt{2}$. Simplify the expression but do NOT evaluate the integral. Sketch the region E.

8. (10pts) Set up and simplify the set-up, but do NOT evaluate the integral: $\int_C \frac{x+y}{xy+1}\, ds$, where C is the part of the curve $y = x^3 - x$ from $(-1, 0)$ to $(1, 0)$.

9. (20pts) Consider the region inside the circle $x^2 + y^2 = 4$ and above the lines $y = \sqrt{3}x$ and $y = -\frac{1}{\sqrt{3}}x$.

a) Draw the region.

b) Use Green's theorem to find the line integral $\int_C y^3\,dx + x^3\,dy$, where C is the boundary of the region D, traversed counterclockwise.

10. (12pts) Use differentials to estimate the change in the volume of a cylindrical can if its height decreases from 28 to 26 centimeters, and its radius increases from 14 to 15 centimeters.

Bonus (10pts) Do problem 6 for the iterated triple integral that ends in $dy\,dz\,dx$.