Calculus 3 — Final Exam
MAT 309, Fall 2016 — D. Ivanšić

Name: ______________________________

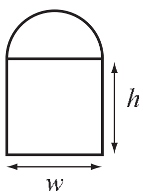
Show all your work!

1. (12pts) Find the equation of the plane whose intersections with the x- and z-axes are 3 and 5, respectively, and that contains the point $(2, 3, -2)$.

2. (22pts) Let $f(x, y) = xy$.
a) Sketch the contour map for the function, drawing level curves for levels $k = -2, -1, 0, 1, 2$.
b) At point $(3, -4)$, find the directional derivative of f in the direction of $\langle 2, -1 \rangle$. In what direction is the directional derivative the greatest? What is the directional derivative in that direction?
c) Let $\mathbf{F} = \nabla f$. Sketch the vector field $\mathbf{F}$.
d) What is $\int_C \mathbf{F} \cdot d\mathbf{r}$ if C is the straight line from $(0, 0)$ to $(5, 2)$?
e) What is $\int_C \mathbf{F} \cdot d\mathbf{r}$ if C is any part of one level curve?

3. (20pts) Find and classify the local extremes for $f(x, y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$.

4. (12pts) A Norman window is a rectangle topped by a semicircle. Use differentials to estimate how much the area of the window changes if the height increases from 52 to 56 inches, and the width decreases from 40 to 38 inches.



5. (18pts) Let D be the region bounded by the curves $y = x^2$, $x = 0$ and $y = 9$. Sketch the region and set up $\displaystyle\iint_D \frac{1}{y^{\frac{3}{2}} + 1}\, dA$ as iterated integrals in both orders of integration. Then evaluate the double integral using the easier order. Sketch the region of integration first.

6. (18pts) Sketch the region E that is bounded by the planes $z = 0$, $x = 9$, $z = 3 - y$ and the parabolic cylinder $x = 9 - y^2$. Then write the two iterated triple integrals that stand for $\iiint_E f \, dV$ which end in $dz\,dy\,dx$ and $dy\,dx\,dz$.

7. (16pts) Use either cylindrical or spherical coordinates to set up $\displaystyle\iiint_E \frac{x+y}{x^2+y^2+z^2}\, dV$, where E is the region inside the sphere $x^2+y^2+z^2 = 16$ and outside the cylinder $x^2+y^2 = 8$. Simplify the expression but do NOT evaluate the integral. Sketch the region E.

8. (10pts) Set up and simplify the set-up, but do NOT evaluate the integral: $\int_C \mathbf{F} \cdot d\mathbf{r}$, if $\mathbf{F}(x, y) = \langle xy, x + y^2 \rangle$, and C is the arc of the circle $x^2 + y^2 = 8$ from point $(-1, \sqrt{7})$ to point $(2, 2)$.

9. (22pts) Use Green's theorem to find the line integral $\int_C (x^2 + y^2)\, dx + xy\, dy$, where C is the boundary of the triangle with vertices $(0, 0)$, $(2, 6)$ and $(0, 12)$, traversed counterclockwise. Draw the triangle.

Bonus (15pts) Let $A = (0, 0)$ and $B = (0, 1)$, and let d_A and d_B represent the distance from a point (x, y) to A and B, respectively. Find the absolute maximum and minimum of $d_A^2 + d_B^2$ among all points (x, y) in the upper half of the unit disk $x^2 + y^2 \leq 1$, $y \geq 0$.