Calculus 3 — Exam 4
MAT 309, Fall 2016 — D. Ivanšić

Name: ____________________

Show all your work!

1. (12pts) Let $\mathbf{F}(x,y) = \left\langle -\frac{x}{2}, -\frac{2y}{9} \right\rangle$.

a) Guess a function $f(x,y)$ so that $\mathbf{F} = \nabla f$.

b) Use the function f to draw the vector field without having to evaluate F at various points.

2. (20pts) In both cases set up and simplify the set-up, but do NOT evaluate the integral.

a) $\int_C \dfrac{y}{x^2+y^2+z^2}\, ds$, where C is the helix $x = 3\cos t$, $y = \frac{t}{\pi}$, $z = 3\sin t$, t in $[0, 4\pi]$.

b) $\int_C \mathbf{F}\cdot d\mathbf{r}$, if $\mathbf{F}(x,y) = \left\langle x^2y^2, \dfrac{x}{y} \right\rangle$, where C is the arc of the hyperbola $x^2 - y^2 = 1$ from point $(2, \sqrt{3})$ to point $(5, 2\sqrt{6})$.

3. (10pts) Let $f(x, y) = x^2 + xy + y^2$, and let $\mathbf{F} = \nabla f$. Apply the fundamental theorem for line integrals to answer:

a) What is $\int_C \mathbf{F} \cdot d\mathbf{r}$ if C is the part of the right half of the circle $x^2 + (y-2)^2 = 4$ from $(0, 0)$ to $(1, 2+\sqrt{3})$? How about if C is a straight line segment from $(0, 0)$ to $(1, 2+\sqrt{3})$?

b) What is $\int_C \mathbf{F} \cdot d\mathbf{r}$ if C is the curve consisting of the left half of the circle from a) together with the line segment from $(0, 0)$ to $(0, 4)$, traversed clockwise?

4. (22pts) a) Find curl of both of the vector fields below.

b) One of the fields is conservative. Find its potential function.

$\mathbf{F}(x, y, z) = \langle \sin z, x, x\cos z - \sin z \rangle$ $\qquad$ $\mathbf{G}(x, y, z) = \langle 3x^2y^2, 2x^3y + e^z, (2+y)e^z \rangle$

5. (24pts) Use Green's theorem to find the line integral $\int_C (x^2 + y^2)\, dx + xy\, dy$, where C is the boundary of the trapezoid with vertices $(1, 0)$, $(3, 0)$, $(3, 7)$ and $(1, 1)$, traversed counterclockwise. Draw the trapezoid.

6. (12pts) Use Green's theorem to find the area of the unit disk.

Bonus. (10pts) Recall that we have shown that the field $\mathbf{F}(x,y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$, defined on the region $D = \mathbf{R}^2$ without the origin, is not conservative on D, since its line integral over a unit circle is not 0.

a) Let $f(x,y) = \arctan \frac{y}{x}$. Show that $\nabla f = \mathbf{F}$. Recall that $(\arctan u)' = \frac{1}{1+u^2}$.

b) Why does a) not contradict our earlier finding of $\mathbf{F}$ not being conservative on D?