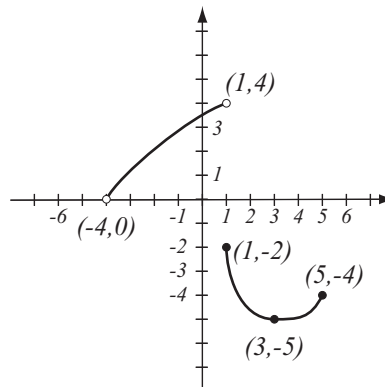


1. (8pts) Let  $A = (-6, 1)$ ,  $B = (-1, -4)$  and  $C = (6, -3)$ . Draw the triangle and then determine algebraically if the triangle  $ABC$  is
- a right triangle,
  - an isosceles triangle (two sides have equal length).

2. (10pts) Write the equation of the circle whose diameter has endpoints  $(-3, 4)$  and  $(1, -2)$ . Sketch the circle.

3. (8pts) Use the graph of the function  $f$  at right to answer the following questions.

- Find  $f(-4)$  and  $f(1)$ .
- What is the domain of  $f$ ?
- What is the range of  $f$ ?
- What are the solutions of the equation  $f(x) = -4$ ?



4. (12pts) The function  $f(x) = x^2 - 6|x| - 4x + 3$  is given.

a) Use your calculator to accurately draw its graph. Draw the graph here, and indicate units on the axes.

b) Find all the  $x$ - and  $y$ -intercepts (accuracy: 6 decimal points).

c) State the domain and range.

5. (12pts) Find the domain of each function and write it using interval notation.

$$f(x) = \frac{\sqrt{x}}{6x - 11}$$

$$g(x) = \frac{x^2 - 5x - 24}{x^2 + 4x - 21}$$

6. (10pts) Let  $g(x) = x^2 + 3x - \sqrt{x}$ . Find the following (simplify where appropriate).

$$g(4) =$$

$$g(-9) =$$

$$g(-t) =$$

$$g(w - 2) =$$



5. (12pts) The water bill for a family was \$52.31 in a month when it used 9 HCF (hundred cubic feet) of water. In another month, it used 13 HCF and was billed \$64.59.

a) Assuming that the water cost  $C(x)$  is a linear function of the amount of water  $x$  used (in HCF), write a formula for  $C(x)$ .

b) What is the cost if no water is used during a month? What is the meaning of this number?

c) What is the meaning of the slope in this example?

6. (20pts) A store is trying to establish the relationship between the price  $P$  of a line of shoes they sell and the weekly sales  $S$  of those shoes. In the table,  $P$  is the price of a pair of shoes in dollars and  $S$  is the number of pairs of shoes sold weekly when the price is as given. Solve the problems below with accuracy 6 decimal points.

a) Draw the scatterplot of the data. Does the relationship look linear?

b) Use two points in the scatterplot to get an equation of a line that models the relationship between  $P$  and  $S$ . Draw the line on the graph.

c) Use your calculator to find the “line of best fit” for the data. Draw the line on the graph.

d) Find the coefficient of correlation  $r$ . How strong is the linear relationship between  $P$  and  $S$ ?

e) What sales can the store expect if they price the shoes at \$94?

$P$	$S$
69	93
79	77
84	66
89	65
99	37
109	33
119	13

Solve the inequalities. Write your solution in interval notation.

1. (5pts)  $-3 \leq 9 - 4x < 5$

2. (7pts)  $3x - 5 < 5$  or  $4x - 6 > 15$

3. (6pts) Find the domain of the function in interval notation:  $f(x) = \frac{\sqrt{5x+8}}{2x-5}$ .

4. (14pts) For her birthday, Christa is considering renting an event venue. Her choices are Party Pad, which charges \$100 per event plus \$40 per hour, or Fiesta Flat, which charges \$200, which includes two hours, and then \$30 per hour for every hour after the first two. Christa plans her party to last at least two hours. For which number of hours is Party Pad the better option for her? Solve as an inequality.

**5.** (14pts) Max rows his boat at 6mph in still water. One day he takes a round-trip on a river, taking 2 hours to row downstream, and then 3 hours to row back upstream.

a) What is the speed of the river?

b) How far did Max travel in one direction?

**6.** (14pts) How many liters of pure water must be mixed with 4 liters of a 20% solution of sulphuric acid in order to get a 13% solution of sulphuric acid?

1. (10pts) Use your calculator to accurately sketch the graph of the function  $f(x) = \frac{5 - 7x}{x^2 + 5x + 9}$ . Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.
- Find the local maxima and minima for this function.
  - State the intervals where the function is increasing and where it is decreasing.

2. (20pts) Let  $f(x) = \frac{x^2 - 3}{x}$ ,  $g(x) = \sqrt{2x + 7}$ . Find the following (simplify where possible):

$$(f - g)(1) = \qquad (fg)(9) =$$

$$\frac{f}{g}(x) = \qquad (g \circ f)(2) =$$

$$(f \circ g)(x) =$$

The domain of  $(f + g)(x)$  in interval notation

3. (8pts) Consider the function  $h(x) = (3\sqrt{x} - 8)^4$  and find **two** different solutions to the following problem: find functions  $f$  and  $g$  so that  $h(x) = f(g(x))$ , where neither  $f$  nor  $g$  are the identity function.

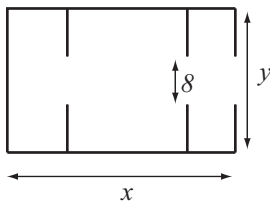
4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 3x + 2, & \text{if } x < -2 \\ -x + 4, & \text{if } -2 \leq x \leq 8. \end{cases}$$

5. (14pts) Maxine is building a small gallery with area 1200 square feet and three rooms with doors 8 feet wide. She wishes to minimize the building cost, which is the same as minimizing the total length of the walls.

a) Express the total length of the walls of the building as a function of the length of one of the sides  $x$ . What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the gallery for which the total length of the walls is minimal? What is the minimal wall length?





College Algebra — Joysheet 5  
MAT 140, Spring 2018 — D. Ivanšić

Name: \_\_\_\_\_

Covers: 2.4, 2.5

Show all your work!

1. (21pts) For the following functions:

a) determine algebraically whether they are odd, even, or neither

b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$$f(x) = x^2 - 4|x| - 5$$

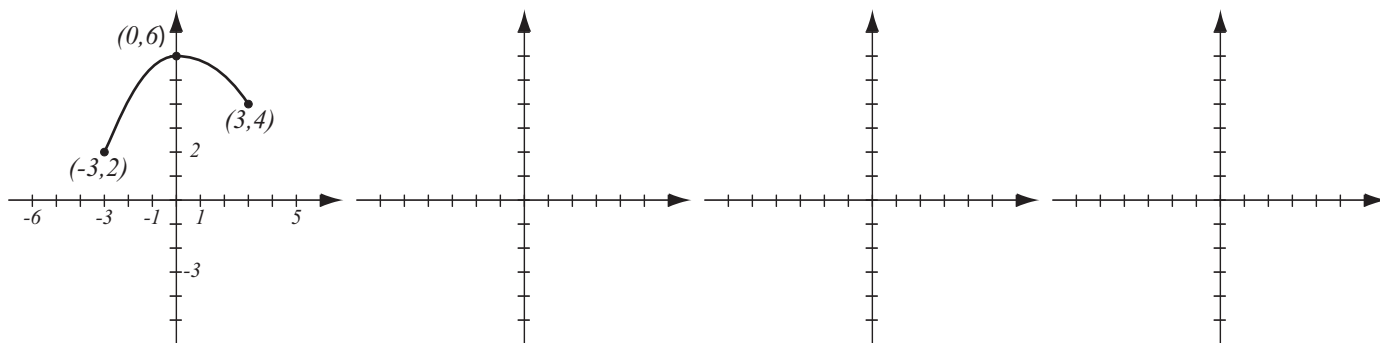
$$g(x) = x^3 - 4x + 5$$

$$h(x) = x^5 + 3x^3 - 7x$$

2. (16pts) Using transformations, draw the graphs of  $f(x) = -(x - 3)^2$  and  $g(x) = 2|x + 4| + 5$ . Explain how you transform graphs of basic functions in order to get the graphs of  $f$  and  $g$ . Indicate at least two points on each graph.

3. (10pts) Write the equation for the function whose graph has the following characteristics:
- shape of  $y = \sqrt[3]{x}$ , shifted up 4 units
  - shape of  $y = x^3$  stretched horizontally by factor 2, then shifted left 4 units
  - shape of  $y = \frac{1}{x}$ , reflected about the  $x$ -axis, then stretched vertically by factor 3, then shifted up 5 units.

4. (13pts) The graph of  $f(x)$  is drawn below. On three separate graphs, sketch the graphs of the functions  $f(x + 2)$ ,  $\frac{1}{2}f(-x)$  and  $f(\frac{1}{2}x) - 3$  and label all the relevant points.



College Algebra — Joysheet 6  
MAT 140, Spring 2018 — D. Ivanšić

Name: \_\_\_\_\_

*Covers: 3.1, 3.2, 3.3 Show all your work!*

Simplify, so that the answer is in form  $a + bi$ .

1. (4pts)  $2i(i + 1) - i(3i + 2) =$

2. (6pts)  $\frac{3 - 2i}{2 - 5i} =$

3. (4pts) Simplify and justify your answer.

$i^{175} =$

4. (8pts) The amount of oil (in tons) arriving daily to refinery is given by  $A(x) = x^2 - 14x + 70$ , where  $x$  is the number of days after March 26th. On what dates were 46 tons arriving daily?

5. (8pts) Solve the equation:  $x^4 - 4x^2 - 45 = 0$

6. (6pts) Solve by completing the square.

$x^2 - 8x - 12 = 0$

7. (12pts) The quadratic function  $f(x) = -2x^2 + 7x - 3$  is given. Do the following without using the calculator.

- a) Find the  $x$ -intercepts of its graph, if any. Find the  $y$ -intercept.
- b) Find the vertex of the graph.
- c) Sketch the graph of the function.

8. (12pts) Farmer Christy has a 3 kilometer by 4 kilometer rectangular field and wishes to enlarge it by increasing the length and the width by the same amount. If the bigger field is to have a diagonal whose length is twice the length of the diagonal of the original field, by how much should she increase the lengths of the sides of the original field?

College Algebra — Joysheet 7  
MAT 140, Spring 2018 — D. Ivanšić

Name: \_\_\_\_\_

*Covers: 3.3, 3.4, 3.5 Show all your work!*

1. (4pts) Solve the equation.

$$|4x + 5| = 1$$

2. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$$|x + 5| < 8$$

$$|3x - 7| < 5$$

Solve the equations:

3. (8pts)  $\frac{2x}{x+1} - \frac{3}{x+4} = \frac{x^2 - 7}{x^2 + 5x + 4}$

4. (8pts)  $x + 8 = 7 + \sqrt{8x + 28}$

5. (14pts) Pig Pen throws a bar of soap upwards with initial velocity 17 meters per second. Its height in meters after  $t$  seconds is given by  $s(t) = -5t^2 + 17t$ .

a) Sketch the graph of the height function.

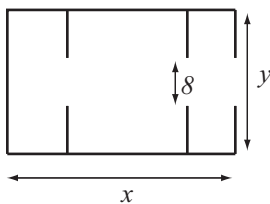
b) When does the soap reach its greatest height, and what is that height?

c) When is the soap at height 6 meters?

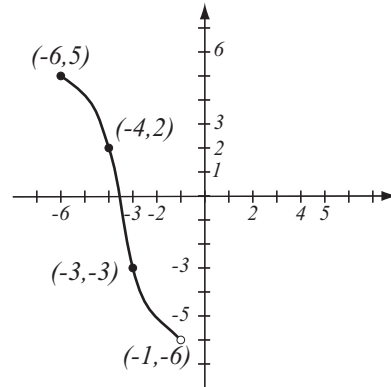
6. (14pts) Maxine is building a small gallery with three rooms that have doors 8 feet wide. She has budgeted for 1500 ft of walls and wishes to maximize the area of the gallery.

a) Express the total area of the gallery as a function of one of the sides of the rectangle. What is the domain of this function?

b) Sketch the graph of the area function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the gallery that has the greatest total area? What is the greatest total area possible?



1. (6pts) The graph of a function  $f$  is given.
- Is this function one-to-one? Justify.
  - If the function is one-to-one, find the graph of  $f^{-1}$ , labeling the relevant points.



2. (12pts) Let  $f(x) = \frac{2x}{3x - 7}$ . Find the formula for  $f^{-1}$ . Find the ranges of  $f$  and  $f^{-1}$ .

3. (8pts) Evaluate without using the calculator:

$$\log_2 32 =$$

$$\log_6 \frac{1}{36} =$$

$$\log_{49} \frac{1}{7} =$$

$$\log \sqrt[3]{b} \sqrt{b} =$$

4. (4pts) Use the change-of-base formula and your calculator to find  $\log_3 33$  with accuracy 6 decimal places. Show how you obtained your number.

5. (12pts) Investigate the effect of increased frequency of compounding: for a deposit of \$10,000 and annual interest rate of 4.26%, calculate the amount in the account after 1 year for the frequencies of compounding below.

- Write the general formula for the amount, replacing the variables by numbers, if known.
- Use the table feature on your calculator to quickly compute amounts after 1 year.
- Does compounding more often make a big difference?

Frequency: every	$n$	Amount after 1 year
year		
quarter		
month		
day		
hour		
second		

6. (3pts) Find the domain of  $f(x) = \ln(7 - 2x)$ .

7. (8pts) An object falling from a height of 300 meters is at height  $h(t) = 300 - 5t^2$  meters after  $t$  seconds.

- Determine the height of the object after 3 and 7 seconds.
- Find a formula for the inverse function and explain what it represents.
- Determine how long the object has been falling its height is 280 and 120 meters.

8. (7pts) Using transformations, draw the graph of  $f(x) = -\ln(x - 5)$ . Explain how you transform the graph of a basic function in order to get the graph of  $f$ . Show at least one point on the graph, and asymptotes to the graph, if any.



1. (5pts) If  $\log_a 5 = u$  and  $\log_a 8 = v$ , express in terms of  $u$  and  $v$ :

$$\log_a \frac{8}{5} =$$

$$\log_a 200 =$$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_4 (64x^3y^{-6}) =$$

$$\log \frac{100x^{12}y^4}{\sqrt{xy^{\frac{7}{2}}}} =$$

3. (12pts) Write as a single logarithm. Simplify if possible.

$$3 \ln(2y^4) - \frac{1}{2} \ln(25x^4) - 6 \ln y =$$

$$3 \log_3(x+2) + 4 \log_3(x-7) - 2 \log_3(x^2 - 5x - 14) =$$

Solve the equations.

4. (5pts)  $49^{x-1} = \left(\frac{1}{7}\right)^{3x-1}$

5. (7pts)  $4^{x+3} = 9^{5x-2}$

6. (8pts)  $2^{2x} - 3 \cdot 2^x - 40 = 0$

7. (12pts) According to US census data, Kentucky had some 4,042,000 inhabitants in 2000 and 4,339,000 in 2010. Assume the population of Kentucky grows exponentially.

a) Write the function describing the number  $P(t)$  of people in Kentucky  $t$  years after 2000.

Then find the exponential growth rate for this population.

b) Graph the function.

c) According to this model, when will the population reach 4,500,000?