College Algebra — Joysheet 9 MAT 140, Spring 2018 — D. Ivanšić	Name: Saul Ocean *Covers: 5.4–5.6* *Show all your work!*

1. (5pts) If $\log_a 5 = u$ and $\log_a 8 = v$, express in terms of u and v:

$\log_a \frac{8}{5} = \log_a 8 - \log_a 5$

$= v - u$

$8 \cdot 5 \cdot 5$

$\log_a 200 = \log_a 8 \cdot 5^2 = \log_a 8 + \log_a 5^2$

$= \log_a 8 + 2\log_a 5$

$= v + 2u$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_4 (64x^3y^{-6}) = \log_4 64 + \log_4 x^3 + \log_4 y^{-6}$

$= 3 + 3\log_4 x - 6\log_4 y$

$x^{\frac{1}{2}}$

$\log \frac{100x^{12}y^4}{\sqrt{x}y^{\frac{7}{2}}} = \log 100 + \log x^{12} + \log y^4 - \log \sqrt{x} - \log y^{\frac{7}{2}}$

$= 2 + 12\log x + 4\log y - \frac{1}{2}\log x - \frac{7}{2}\log y$

$= 2 + \frac{23}{2}\log x + \frac{1}{2}\log y$

3. (12pts) Write as a single logarithm. Simplify if possible.

$3\ln(2y^4) - \frac{1}{2}\ln(25x^4) - 6\ln y = \ln(2y^4)^3 - \ln(25x^4)^{\frac{1}{2}} - \ln y^6$

$= \ln \frac{8y^{12}}{5x^2 \cdot y^6} = \ln \frac{8y^6}{5x^2}$

$3\log_3(x+2) + 4\log_3(x-7) - 2\log_3(x^2-5x-14) = \log_3 (x+2)^3 + \log_3 (x-7)^4 - \log_3(x^2-5x-14)^2$

$(x+2)(x-7)$

$= \log_3 \frac{(x+2)^{\cancel{3}\,1}(x-7)^{\cancel{4}\,2}}{\cancel{(x+2)^2}\,\cancel{(x-7)^2}} = \log_3\left((x+2)(x-7)^2\right)$

Solve the equations.

4. (5pts) $49^{x-1} = \left(\frac{1}{7}\right)^{3x-1}$

$(7^2)^{x-1} = (7^{-1})^{3x-1}$

$7^{2x-2} = 7^{-3x+1}$

$2x-2 = -3x+1$

$5x = 3$

$x = \frac{3}{5}$

5. (7pts) $4^{x+3} = 9^{5x-2}$ $\mid \ln$

$\ln 4^{x+3} = \ln 9^{5x-2}$

$(x+3)\ln 4 = (5x-2)\ln 9$

$x\ln 4 + 3\ln 4 = 5x\ln 9 - 2\ln 9$

$x\ln 4 - x\cdot 5\ln 9 = -3\ln 4 - 2\ln 9$

$x(\ln 4 - 5\ln 9) = -3\ln 4 - 2\ln 9$

$x = \frac{-3\ln 4 - 2\ln 9}{\ln 4 - 5\ln 9} = \frac{3\ln 4 + 2\ln 9}{5\ln 9 - \ln 4} =$

$= 0.890988$

6. (8pts) $2^{2x} - 3\cdot 2^x - 40 = 0$

$(2^x)^2 - 3\cdot 2^x - 40 = 0$ $\quad u = 2^x$

$u^2 - 3u - 40 = 0$

$(u-8)(u+5) = 0$

$u = 8, -5$

$2^x = 8$ or $2^x = -5$

$x = 3$

no solution since $2^x > 0$ always

7. (12pts) According to US census data, Kentucky had some 4,042,000 inhabitants in 2000 and 4,339,000 in 2010. Assume the population of Kentucky grows exponentially.
a) Write the function describing the number $P(t)$ of people in Kentucky t years after 2000. Then find the exponential growth rate for this population.
b) Graph the function.
c) According to this model, when will the population reach 4,500,000?

a) $P(t) = 4042e^{kt}$ (in thousands)

$P(10) = 4339$

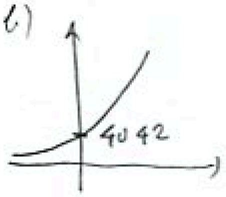
$4339 = 4042e^{k\cdot 10}$

$\frac{4339}{4042} = e^{10k}$ $\mid \ln$

$\ln\frac{4339}{4042} = 10k$

$k = \frac{\ln\frac{4339}{4042}}{10} = 0.00705093$

b)



c) $4500 = 4042e^{0.00705093t}$

$\frac{4500}{4042} = e^{0.007\ldots t}$ $\mid \ln$

$\ln\frac{4500}{4042} = 0.007\ldots t$

$t = \frac{\ln\frac{4500}{4042}}{0.00705093} = 15.138404$

t years after 2000

In about 2015.