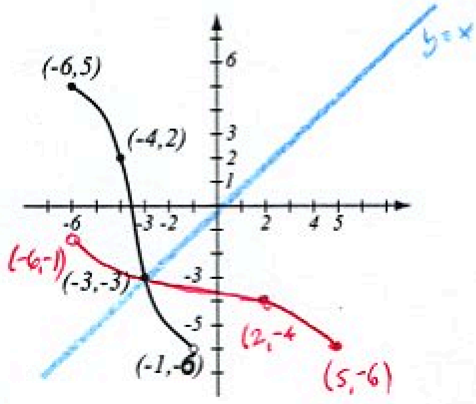
College Algebra — Joysheet 8
MAT 140, Spring 2018 — D. Ivanšić

Name: Saul Ocean

Covers: 5.1–5.3 *Show all your work!*

1. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.



a) yes - passes horizontal line test

b) reflect in line $y=x$
(effect on coordinates: swap x and y)

2. (12pts) Let $f(x) = \dfrac{2x}{3x-7}$. Find the formula for f^{-1}. Find the ranges of f and f^{-1}.

$y = \dfrac{2x}{3x-7}$

$(3x-7)y = 2x$

$3xy - 7y = 2x$

$3xy - 2x = 7y$

$x(3y-2) = 7y$

$x = \dfrac{7y}{3y-2}$

$f^{-1}(y) = \dfrac{7y}{3y-2}$

Range of f = domain of $f^{-1}(y) = \dfrac{7y}{3y-2}$

Can't have $3y-2=0$

$3y=2$

$y = \frac{2}{3}$ $\frac{2}{3}$

Range $f = (-\infty, \frac{2}{3}) \cup (\frac{2}{3}, \infty)$

Range of f^{-1} = domain of $f(x) = \dfrac{2x}{3x-7}$

Can't have $3x-7=0$

$3x=7$

$x = \frac{7}{3}$ $\frac{7}{3}$

$(-\infty, \frac{7}{3}) \cup (\frac{7}{3}, \infty)$ Range f^{-1}

3. (8pts) Evaluate without using the calculator:

$\log_2 32 = 5$

$2^{?} = 32$

$\log_6 \frac{1}{36} = -2$

$6^{?} = \frac{1}{36} = \frac{1}{6^2} = 6^{-2}$

$\log_{49} \frac{1}{7} = -\frac{1}{2}$

$49^{?} = \frac{1}{7} = \frac{1}{\sqrt{49}} = \frac{1}{49^{\frac{1}{2}}} = 49^{-\frac{1}{2}}$

$\log_{\sqrt[3]{b}} \sqrt{b} = \frac{3}{2}$

$(\sqrt[3]{b})^{?} = \sqrt{b}$

$(b^{\frac{1}{3}})^{?} = b^{\frac{1}{2}}$

$\frac{1}{3} \cdot ? = \frac{1}{2}$ 1.3

$? = \frac{3}{2}$

4. (4pts) Use the change-of-base formula and your calculator to find $\log_3 33$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_3 33 = \dfrac{\ln 33}{\ln 3} = 3.182658$

5. (12pts) Investigate the effect of increased frequency of compounding: for a deposit of \$10,000 and annual interest rate of 4.26%, calculate the amount in the account after 1 year for the frequencies of compounding below.
a) Write the general formula for the amount, replacing the variables by numbers, if known.
b) Use the table feature on your calculator to quickly compute amounts after 1 year.
c) Does compounding more often make a big difference?

Frequency: every	n	Amount after 1 year
b) year	1	10,426
quarter	4	10,432.85
month	12	10,434.42
day	365	10,435.18
hour	365·24	10,435.20
second	365·24·3600	10,435.19

a) $A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 10000\left(1+\frac{0{,}0426}{n}\right)^{n\cdot 1}$

c) Compounding more often than monthly makes little difference.

6. (3pts) Find the domain of $f(x) = \ln(7 - 2x)$.

Must have: $7-2x>0$

$7>2x$

$\frac{7}{2}>x$

$\frac{7}{2}$

$(-\infty, \frac{7}{2})$

7. (8pts) An object falling from a height of 300 meters is at height $h(t) = 300 - 5t^2$ meters after t seconds.
a) Determine the height of the object after 3 and 7 seconds.
b) Find a formula for the inverse function and explain what it represents.
c) Determine how long the object has been falling its height is 280 and 120 meters.

a) $h(3) = 300 - 5\cdot 3^2 = 255$

$h(7) = 300 - 5\cdot 7^2 = 55$

b) $h = 300 - 5t^2$

$5t^2 = 300 - h$

$t^2 = \frac{300-h}{5}$

$t = \pm\sqrt{\frac{300-h}{5}}$

$t = \sqrt{\frac{300-h}{5}}$ since $t>0$

Represents time it takes for an object to fall to height h.

c) $t(280) = \sqrt{\frac{300-280}{5}} = \sqrt{4} = 2$ seconds

$t(120) = \sqrt{\frac{300-120}{5}} = \sqrt{\frac{180}{5}} = \sqrt{36} = 6$ sec.

8. (7pts) Using transformations, draw the graph of $f(x) = -\ln(x-5)$. Explain how you transform the graph of a basic function in order to get the graph of f. Show at least one point on the graph, and asymptotes to the graph, if any.