College Algebra — Joysheet 6
MAT 140, Spring 2018 — D. Ivanšić

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Covers: 3.1, 3.2, 3.3 Show all your work!

Simplify, so that the answer is in form $a+bi$.

1. (4pts) $2i(i+1)-i(3i+2) =$ $2i^2+2i-3i^2-2i$ ($i^2=-1$)

$= -2-(-3) = 1$

2. (6pts) $\frac{3-2i}{2-5i} = \frac{3-2i}{2-5i}\cdot\frac{2+5i}{2+5i} = \frac{6+15i-4i-10i^2}{2^2-(5i)^2} = \frac{16+11i}{29}$ ($i^2=-1$)

$4-(-25)$

3. (4pts) Simplify and justify your answer.

$i^{175} = i^{172}\cdot i^3 = (i^4)^{43}\cdot i^3 = 1\cdot i^3 = -i$ ($i^4=1$)

$175 = 4\cdot 43$

4. (8pts) The amount of oil (in tons) arriving daily to refinery is given by $A(x) = x^2 - 14x + 70$, where x is the number of days after March 26th. On what dates were 46 tons arriving daily?

$x^2-14x+70 = 46 \quad |-46$

$x^2-14x+24=0$

$(x-2)(x-12)=0$

$x = 2, 12$

On March 28th or April 7th

5. (8pts) Solve the equation: $x^4-4x^2-45=0$ Let $u=x^2$

$u^2-4u-45=0$

$(u+5)(u-9)=0$

$u=-5, 9$

$x^2=-5$ or $x^2=9$

$x=\pm\sqrt{5}i \quad x=\pm 3$

6. (6pts) Solve by completing the square.

$x^2-8x-12=0$

$x^2-2\cdot x\cdot 4 = 12 \quad |+4^2$

$x^2-2x\cdot 4+4^2 = 12+4^2$

$(x-4)^2 = 28$

$x-4 = \pm\sqrt{28} = \pm 2\sqrt{7}$ (4.7)

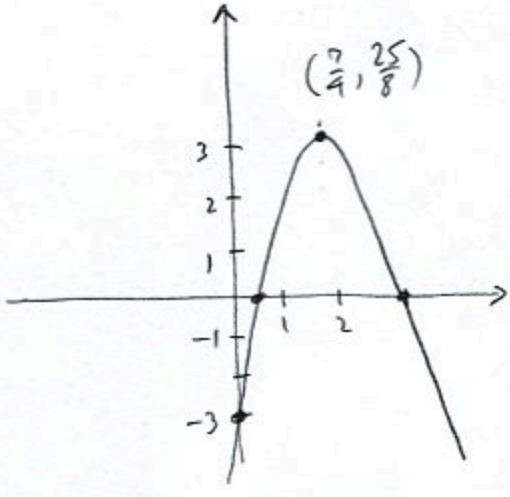
$x = 4\pm 2\sqrt{7}$

7. (12pts) The quadratic function $f(x) = -2x^2 + 7x - 3$ is given. Do the following without using the calculator.

a) Find the x-intercepts of its graph, if any. Find the y-intercept.

b) Find the vertex of the graph.

c) Sketch the graph of the function.



a) y-int: $f(0) = -3$

x-int: $-2x^2+7x-3=0$

$2x^2-7x+3=0$

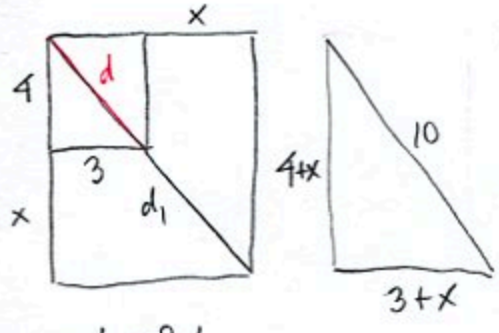
$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4\cdot 2\cdot 3}}{2\cdot 2}$

$= \dfrac{7 \pm \sqrt{25}}{4} = \dfrac{7\pm 5}{4} = 3, \dfrac{1}{2}$

b) $h = -\dfrac{b}{2a} = -\dfrac{7}{2\cdot(-2)} = \dfrac{7}{4}$

$k = f\left(\frac{7}{4}\right) = -2\left(\frac{7}{4}\right)^2 + 7\cdot\frac{7}{4} - 3 = -2\cdot\frac{49}{16} + \frac{49}{4} - 3 = -\frac{49}{8} + \frac{49}{4} - 3 = \dfrac{-49+98-24}{8} = \dfrac{25}{8} = 3\frac{1}{8}$

8. (12pts) Farmer Christy has a 3 kilometer by 4 kilometer rectangular field and wishes to enlarge it by increasing the length and the width by the same amount. If the bigger field is to have a diagonal whose length is twice the length of the diagonal of the original field, by how much should she increase the lengths of the sides of the original field?



$d_1 = 2d$

$d^2 = 3^2 + 4^2$

$d^2 = 25$

$d = \pm 5$

$d = 5$, since $d \geq 0$

Thus $d_1 = 10$

$(4+x)^2 + (3+x)^2 = 10^2$ by Pythagorean thm

$16 + 8x + x^2 + 9 + 6x + x^2 = 100$

$2x^2 + 14x - 75 = 0$

$196+600$

$x = \dfrac{-14 \pm \sqrt{14^2 - 4\cdot 2\cdot(-75)}}{2\cdot 2} = \dfrac{-14 \pm \sqrt{796}}{4}$

$= \dfrac{-14 \pm \sqrt{4\cdot 199}}{4} = \dfrac{-14 \pm 2\sqrt{199}}{4} = \dfrac{2(-7 \pm \sqrt{199})}{4}$

$= \dfrac{-7 \pm \sqrt{199}}{2}$ since $\dfrac{-7-\sqrt{199}}{2} < 0$, only

$x = \dfrac{-7+\sqrt{199}}{2} \approx 3.553368$ is a solution