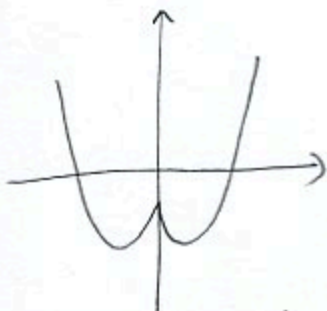**1.** (21pts) For the following functions:
a) determine algebraically whether they are odd, even, or neither
b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$f(x) = x^2 - 4|x| - 5$

$f(-x) = (-x)^2 - 4|-x| - 5$
$= x^2 - 4|x| - 5 = f(x)$

so even

$g(x) = x^3 - 4x + 5$

$g(-x) = (-x)^3 - 4(-x) + 5$
$= -x^3 + 4x + 5 \begin{array}{l} \neq f(x) \\ \neq -f(x) \end{array}$

so neither

$h(x) = x^5 + 3x^3 - 7x$

$h(-x) = (-x)^5 + 3(-x)^3 - 7(-x)$
$= -x^5 - 3x^3 + 7x = -f(x)$

so odd

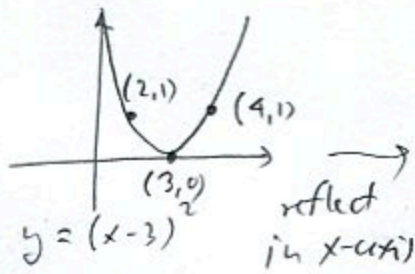symmetric wrt
y-axis
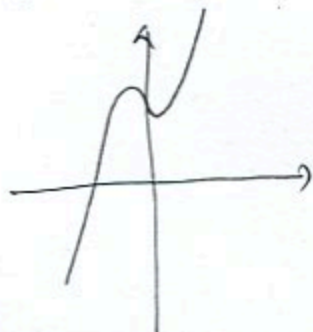
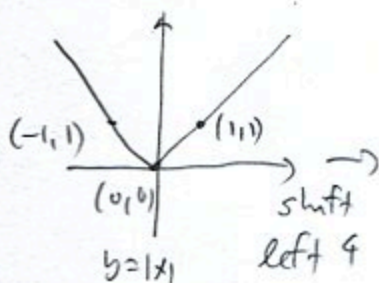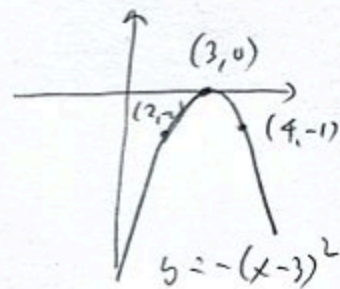no symmetry
wrt origin or y-axis
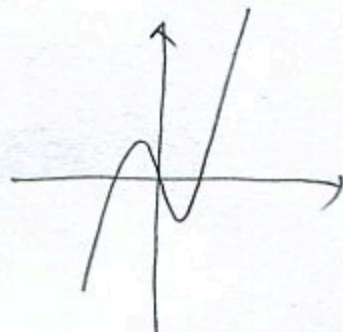
symmetric wrt origin

2. (16pts) Using transformations, draw the graphs of $f(x) = -(x-3)^2$ and $g(x) = 2|x+4| + 5$. Explain how you transform graphs of basic functions in order to get the graphs of f and g. Indicate at least two points on each graph.
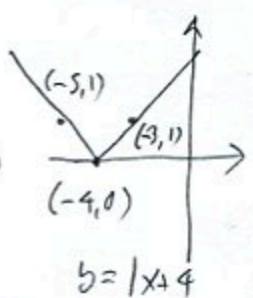

$(-1,1)$ $(1,1)$
$(0,0)$
$y = x^2$

shift right 3

$(2,1)$ $(4,1)$
$(3,0)$
$y = (x-3)^2$

reflect in x-axis

$(3,0)$
$(2,-1)$ $(4,-1)$
$y = -(x-3)^2$

$(-1,1)$ $(1,1)$
$(0,0)$
$y = |x|$
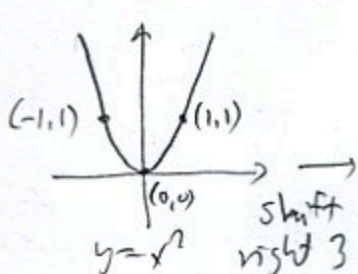
shift left 4

$(-5,1)$ $(-3,1)$
$(-4,0)$
$y = |x+4|$

stretch vertically, factor = 2

$(-5,2)$ $(-3,2)$
$(-4,0)$
$y = 2|x+4|$
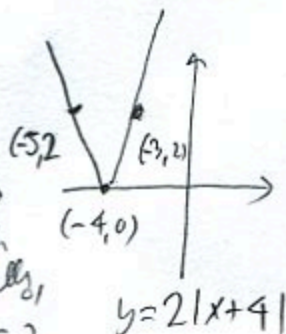
shift up 5

$(-5,7)$ $(-3,7)$
$(-4,5)$
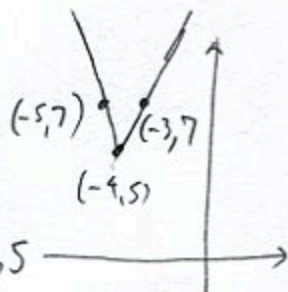
3. (10pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = \sqrt[3]{x}$, shifted up 4 units
b) shape of $y = x^3$ stretched horizontally by factor 2, then shifted left 4 units
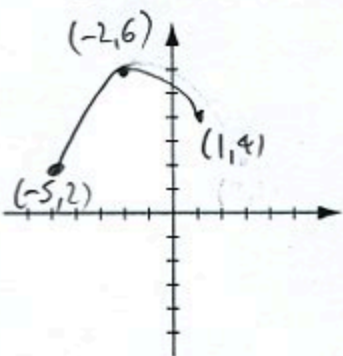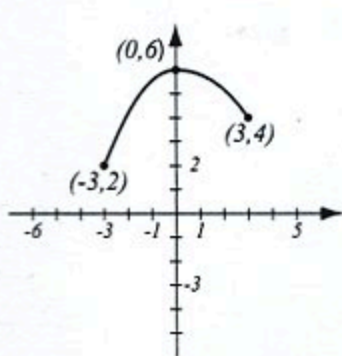c) shape of $y = \frac{1}{x}$, reflected about the x-axis, then stretched vertically by factor 3, then shifted up 5 units.
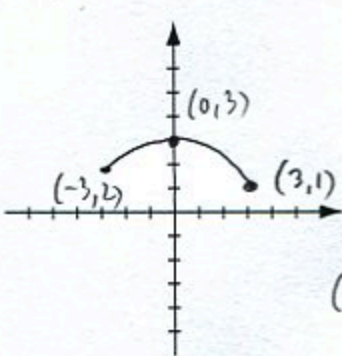
a) $y = \sqrt[3]{x} + 4$

b) $y = x^3 \longrightarrow y = \left(\frac{1}{2}x\right)^3 \longrightarrow y = \left(\frac{1}{2}(x+4)\right)^3$

c) $y = \frac{1}{x} \longrightarrow y = -\frac{1}{x} \longrightarrow y = -\frac{3}{x} \longrightarrow y = -\frac{3}{x} + 5$
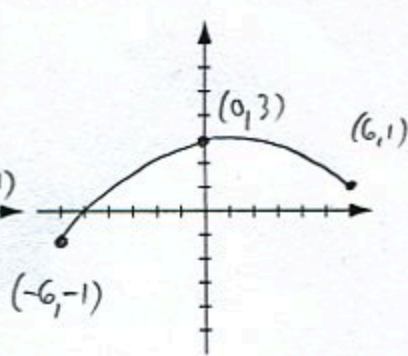
4. (13pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x+2)$, $\frac{1}{2}f(-x)$ and $f(\frac{1}{2}x) - 3$ and label all the relevant points.

(0,6)   (3,4)   (-3,2)   2   -6   -3   -1   1   5   -3

(-2,6)   (1,4)   (-5,2)   shift left 2

(0,3)   (-3,2)   (3,1)   reflect in y-axis, stretch vertically, factor = $\frac{1}{2}$

(0,3)   (6,1)   (-6,-1)   stretch horizontally, factor = 2   shift down 3