College Algebra — Joysheet 4
MAT 140, Spring 2018 — D. Ivanšić

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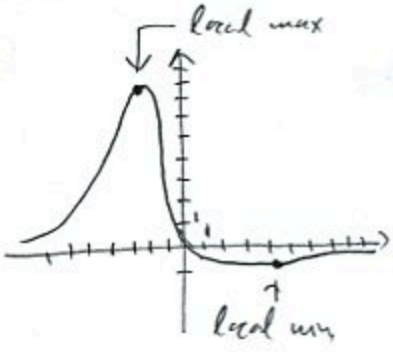
Covers: 2.1, 2.2, 2.3 Show all your work!

1. (10pts) Use your calculator to accurately sketch the graph of the function $f(x) = \dfrac{5-7x}{x^2+5x+9}$. Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.

a) Find the local maxima and minima for this function.

b) State the intervals where the function is increasing and where it is decreasing.



a) $f(-2.90257) = 8.694178$ is a local max

$f(4.331141) = -0.51236$ is a local min

b) Increasing on $(-\infty, -2.90257)$ and $(4.331141, \infty)$

Decreasing on $(-2.90257, 4.331141)$

2. (20pts) Let $f(x) = \dfrac{x^2-3}{x}$, $g(x) = \sqrt{2x+7}$. Find the following (simplify where possible):

$(f-g)(1) = f(1) - g(1) = \dfrac{1-3}{1} - \sqrt{2\cdot 1+7}$

$= -2 - 3 = \boxed{-5}$

$(fg)(9) = f(9)\cdot g(9) = \dfrac{81-3}{9}\cdot\sqrt{2\cdot 9+7}$

$= \dfrac{26}{3}\cdot\sqrt{25} = \dfrac{26\cdot 5}{3} = \boxed{\dfrac{130}{3}}$

$\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{\frac{x^2-3}{x}}{\sqrt{2x+7}} = \dfrac{x^2-3}{x}\cdot\dfrac{1}{\sqrt{2x+7}}$

$= \boxed{\dfrac{x^2-3}{x\sqrt{2x+7}}}$

$(g\circ f)(2) = g(f(2)) = g\left(\dfrac{2^2-3}{2}\right) = g\left(\dfrac{1}{2}\right)$

$= \sqrt{2\cdot\frac{1}{2}+7} = \sqrt{8} = \boxed{2\sqrt{2}}$

$(f\circ g)(x) = f(g(x)) = f\left(\sqrt{2x+7}\right) = \dfrac{\left(\sqrt{2x+7}\right)^2-3}{\sqrt{2x+7}} = \dfrac{2x+7-3}{\sqrt{2x+7}} = \boxed{\dfrac{2x+4}{\sqrt{2x+7}}}$

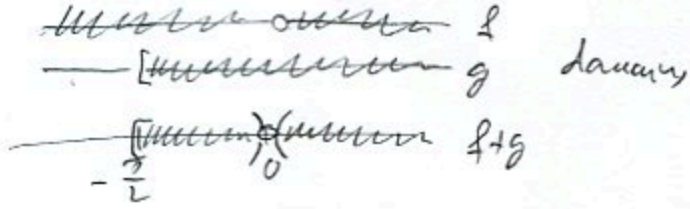
The domain of $(f+g)(x)$ in interval notation

domain f: can't have $x=0$

domain g: must have $2x+7 \geq 0$

$2x \geq -7$

$x \geq -\frac{7}{2}$



$\left[-\frac{7}{2}, 0\right) \cup (0, \infty)$

3. (8pts) Consider the function $h(x) = (3\sqrt{x} - 8)^4$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

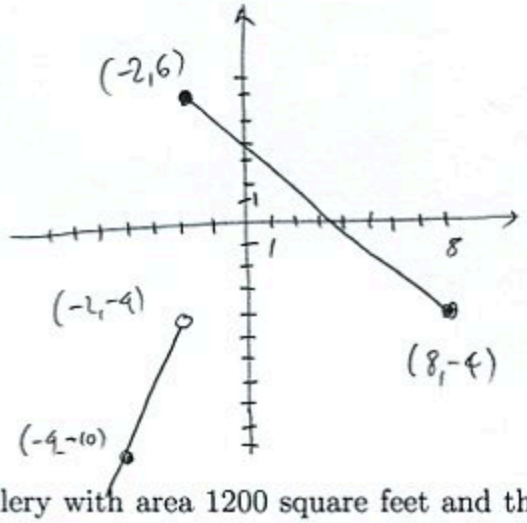
$g(x) = 3\sqrt{x} - 8$ $\quad f(x) = x^4$ or $g(x) = 3\sqrt{x}$ $\quad f(x) = (x-8)^2$ or $g(x) = \sqrt{x}$ $\quad f(x) = (3x-8)^4$

4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 3x + 2, & \text{if } x < -2 \\ -x + 4, & \text{if } -2 \le x \le 8. \end{cases}$$

x	$3x+2$
-2	-4
-4	-10

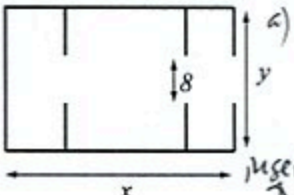
x	$-x+4$
-2	6
8	-4



5. (14pts) Maxine is building a small gallery with area 1200 square feet and three rooms with doors 8 feet wide. She wishes to minimize the building cost, which is the same as minimizing the total length of the walls.

a) Express the total length of the walls of the building as a function of the length of one of the sides x. What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the gallery for which the total length of the walls is minimal? What is the minimal wall length?



a) $\ell = 2x + y + 3(y-8)$

$= 2x + 4y - 24$

$= 2x + \frac{4 \cdot 1200}{x} - 24$ insert

$xy = 1200$

$y = \frac{1200}{x}$

$\ell(x) = 2x + \frac{4800}{x} - 24$

Must have $x \ge 0$, $x \ne 0$ Domain: $(0, 150]$

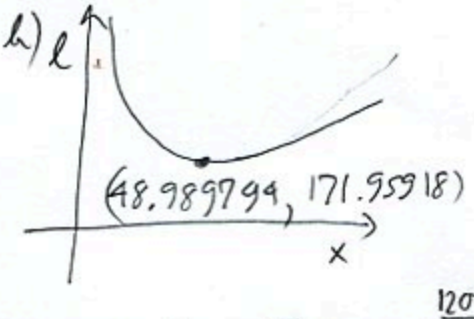
$y \ge 8$

$\frac{1200}{x} \ge 8$

$1200 \ge 8x$

$\frac{1200}{8} \ge x$, $x \le 150$

b)



Dimensions of gallery: $\frac{1200}{48.989794}$

$48.989794 \times 24.494898$ ft

Minimal length of walls:

171.95918 ft