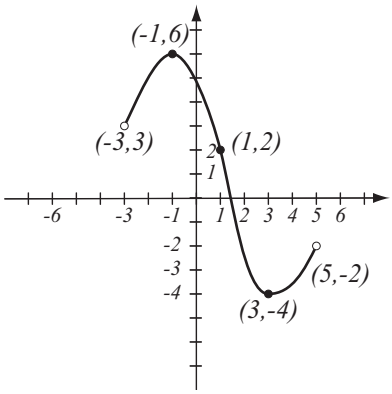
College Algebra — Exam 1
MAT 140, Spring 2018 — D. Ivanšić

Name: _____

Show all your work!

1. (8pts) Use the graph of the function f at right to answer the following questions.



a) Find: $f(3) =$ $f(-3) =$

b) What is the domain of f?

c) What is the range of f?

d) What are the solutions of the equation $f(x) = -3$?

2. (10pts) Use your calculator to accurately sketch the graph of $y = x^4 - 5x^2 - 10$.

a) Draw the graph on paper and indicate units on the axes.

b) Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).

3. (5pts) Write the equation of the line that passes through points $(2, -3)$ and $(-4, 9)$.

4. (10pts) Find the equation of the line (in form $y = mx + b$) that is parallel to the line $3x - 2y = 8$ and has x-intercept 5. Draw both lines.

5. (7pts) Draw the triangle with vertices $A = (-3, 4)$, $B = (2, -1)$, $C = (3, 2)$. Use either slopes or lengths of sides (distance formula) to determine whether the triangle is a right triangle.

6. (9pts) Let $f(x) = \dfrac{\sqrt{2x-7}}{x^2 - 3x + 2}$. Find the following (simplify where appropriate).

$f(8) =$ $f(2) =$

$f(-2x) =$ $f(u-3) =$

7. (9pts) Find the domains of the functions below and write them using interval notation.

$f(x) = \dfrac{3x-1}{x^2 + 4x - 5}$ $g(x) = \sqrt{16 - 6x}$

8. (6pts) Solve and write the solution in interval notation.

$3x + 1 < 4$ or $2x - 5 > 9$

9. (10pts) The endpoints of a diameter of a circle are $(-1, 5)$ and $(3, -1)$.
a) Find the equation of the circle.
b) Draw the circle in the coordinate plane.

10. (12pts) Zoe is considering which taxi company to use for a trip:
Ridewith charges a \$4.00 for any ride up to one mile plus \$1.75 per mile for miles past one.
Tripbuddy charges \$2.25 per mile.
If Zoe rides more than one mile, for which number of miles traveled is Tripbuddy the better option? Solve as an inequality.

11. (14pts) Shepherd Billy looked away from his cow just when it started trotting away at 4 meters per second. Having realized what is happening 12 seconds later, he starts to chase the cow, running at 7 meters per second.
a) How long does Billy run until he catches up with the cow?
b) How far does he run until that moment?

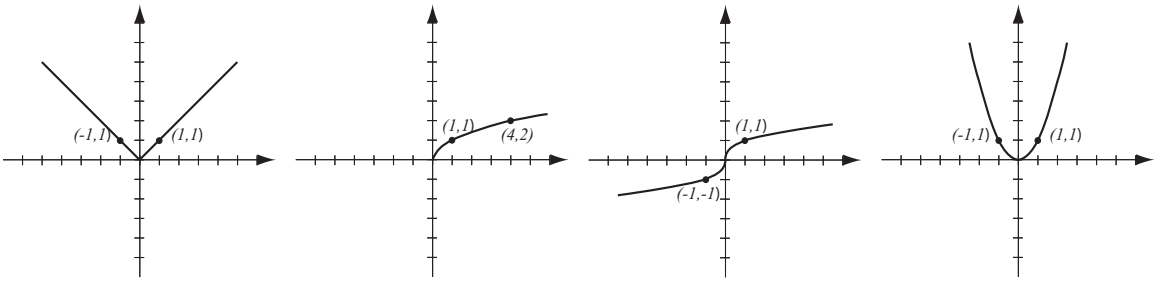
Bonus (10pts) A university invests 1,400,000 at simple interest, part at 5%, half that amount at 3.5% and the rest at 5.5%. What is the most that the university can invest at 3.5% and still have at least $68,000 in interest per year? Solve as an inequality.

College Algebra — Exam 2
MAT 140, Spring 2018 — D. Ivanšić

Name: _____

Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (21pts) Let $f(x) = \dfrac{3x-2}{x+1}$, $g(x) = \dfrac{1}{x-4}$.

Find the following (simplify where possible):

$(f+g)(1) =$ $(fg)(-2) =$

$\dfrac{f}{g}(x) =$ $(f \circ g)(5) =$

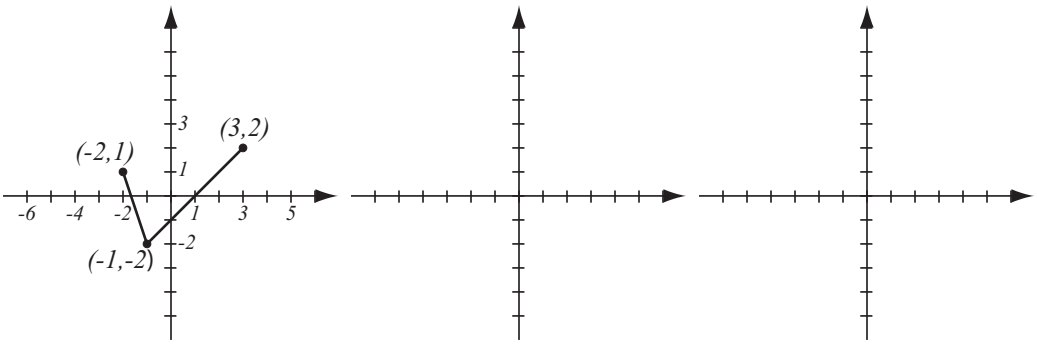
$(g \circ f)(x) =$

The domain of $f - g$ in interval notation

3. (6pts) Consider the function $h(x) = 5(x+1)^2$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

4. (6pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = \sqrt{x}$, shifted right 4 units
b) shape of $y = x^2$, stretched horizontally by factor 3, then shifted down 5 units.

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x+3)$ and $2f(-x)$ and label all the relevant points.



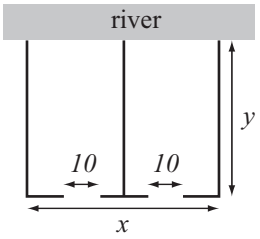
6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x+3, & \text{if } -4 \le x < 0 \\ x-6, & \text{if } 0 \le x \le 5 \end{cases}$$

7. (7pts) For the function $f(x) = x^4 - 6x^2 + 5$:
a) Determine algebraically whether it is odd, even, or neither.
b) Use the calculator to sketch its graph here and verify your conclusion by stating symmetry.

8. (20pts) Let $f(x) = x^3 - 12x$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.
b) Determine algebraically whether the function is odd, even, or neither.
c) Verify your conclusion from b) by stating symmetry.
d) Find the local maxima and minima for this function.
e) State the intervals where the function is increasing and where it is decreasing.

9. (14pts) Farmer Joe is fencing a rectangular field next to a river, dividing it in two sections and leaving a 10-foot opening in each section. The side along the river does not require fencing. The field must have area 6,000 square feet and Joe's goal is to minimize the total length of the fence.
a) Express the total fence length as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the field that has the smallest total fence length and what is the minimal fence length?



Bonus. (10pts) Consider all lines through point $(2, 1)$ with **negative** slope.
a) Draw one such line. With the axes, it forms a triangle in the first quadrant whose area depends on the slope m.
b) Write the equation of a line through $(2, 1)$ with slope m (the equation will have m in it).
c) Determine the x- and y-intercepts of the line in b). They will depend on m.
d) Using c), write the expression for the area of the triangle described in a). It will depend only on m.

College Algebra — Exam 3
MAT 140, Spring 2018 — D. Ivanšić

Name: _____

Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (5pts) $i(3 - i) + 3(1 - 2i) =$

2. (5pts) $\dfrac{2 - i}{1 + 3i} =$

3. (4pts) Simplify and justify your answer.

$i^{218} =$

4. (6pts) Solve the equation by completing the square.

$x^2 - 14x + 17 = 0$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|x - 4| \geq 7$

6. (6pts) Let $P(x)$ be a polynomial of degree 5.
a) What is the maximal number of x-intercepts that $P(x)$ can have? The maximal number of turning points?
b) Draw a graph of P that has the maximal number of x-intercepts and turning points.
c) Draw a graph of P that has exactly 3 x-intercepts and 4 turning points.

7. (12pts) The quadratic function $f(x) = x^2 - 2x + 5$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

Solve the equations:

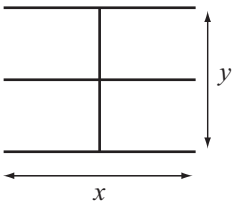
8. (8pts) $\dfrac{x}{x+1} + \dfrac{16}{x^2-6x-7} = \dfrac{2}{x-7}$

9. (8pts) $1 - \sqrt{19+6x} = x + 3$

10. (14pts) The polynomial $f(x) = (x-5)(x+2)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

11. (12pts) In a right triangle, the hypothenuse is twice the length of the shorter side, and the longer side is 3 centimeters longer than the shorter side. What is the length of the shorter side in this right triangle?

12. (14pts) A local businesswoman is building a repair shop with 4 bays, as in the picture. She has enough money to build 300 feet of walls, and her goal is to maximize the total area of the shop.
a) Express the total area of the shop as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the shop that has the biggest possible total area, and what is the biggest possible total area?



Bonus. (10pts) Verify, by plugging in and doing the algebra, that all of the numbers $-2i$, $\sqrt{3}+i$ and $-\sqrt{3}+i$ are solutions of the equation $z^3 = 8i$. One is easy and two are not so easy. (This is an illustration of a general fact: every equation $z^3 = b$ has three solutions among complex numbers.)

College Algebra — Exam 4
MAT 140, Spring 2018 — D. Ivanšić

Name: _____

Show all your work!

1. (8pts) Evaluate without using the calculator:

$\log_3 81 =$ $\log_2 \frac{1}{16} =$ $\log_a \sqrt{a^7} =$ $\log_{b^3} b^{12} =$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_7 0.56$ with accuracy 6 decimal places. Show how you obtained your number.

3. (5pts) If $\log_a 5 = u$ and $\log_a 4 = v$, express in terms of u and v:

$\log_a 20 =$ $\log_a \dfrac{5}{16} =$

4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_7 \dfrac{y^4}{49\sqrt[3]{x^4}} =$

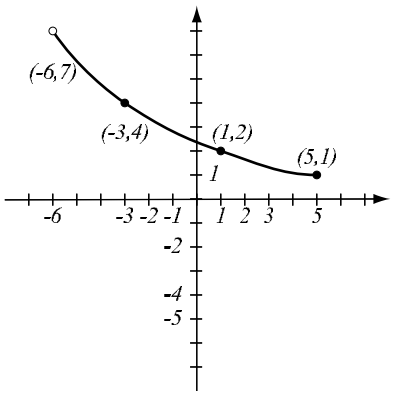
5. (6pts) Write as a single logarithm. Simplify if possible.

$3\log(x^2y^{-3}) - 2\log(x^4y) =$

6. (4pts) Simplify.

$\ln e^{3x-4} =$ $6^{\log_6 \sqrt{2}} =$

7. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points, and showing any asymptotes.



8. (9pts) Let $f(x) = \dfrac{x-3}{4x}$.
a) Find the formula for f^{-1}.
b) Find the range of f.

9. (6pts) Using transformations, draw the graph of $f(x) = 4 + e^{-x}$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.

10. (3pts) Find the domain of the function $f(x) = \log_5(4x + 9)$ and write it in interval notation.

11. (9pts) What is better: an account bearing 5.1% compounded monthly, or an account bearing 5.2% compounded quarterly? Find out by comparing \$100 deposits placed for a year.

Solve the equations.

12. (6pts) $2^{2x-1} = 8^{x-3}$

13. (8pts) $5^{x+3} = 9^{2x}$

14. (8pts) $\log_2(2x - 3) - \log_2(x - 7) = 2$

15. (12pts) The population of Breedington was 12,000 in 2011 and 14,000 in 2015. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2011. Graph it on paper.
b) Find the predicted population in the year 2020.

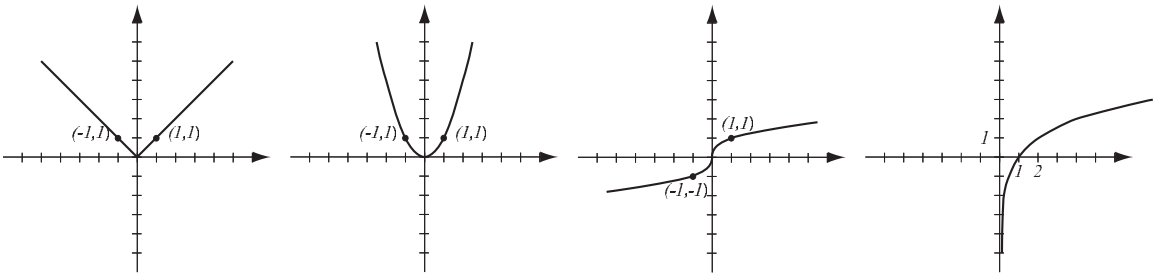
Bonus (10pts) Let $f(x) = x^2 - 2x$, with domain $x \geq 1$.
a) Graph the function (sketch on paper!). Explain why it is one-to-one.
b) Find the formula for $f^{-1}(x)$. (Once you set it up, solving for x involves doing a quadratic equation, which you solve using the quadratic formula.)

College Algebra — Final Exam
MAT 140, Spring 2018 — D. Ivanšić

Name: _____

Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



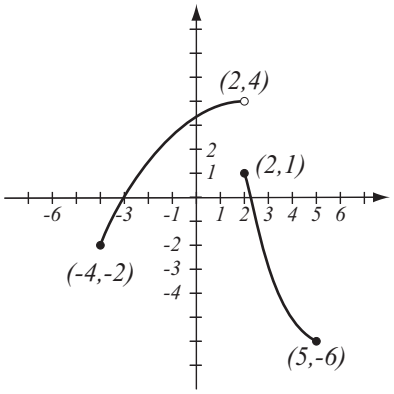
2. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find: $f(-3) =$ $f(2) =$

b) What is the domain of f?

c) What is the range of f?

d) What are the solutions of the equation $f(x) = -2$?



3. (12pts) a) Write the equation of the line whose y-intercept is 2 and has slope 3.
b) Write the equation of the line through points $(-1, 3)$ and $(2, 2)$.
c) Are the two lines perpendicular?
d) Draw both lines.

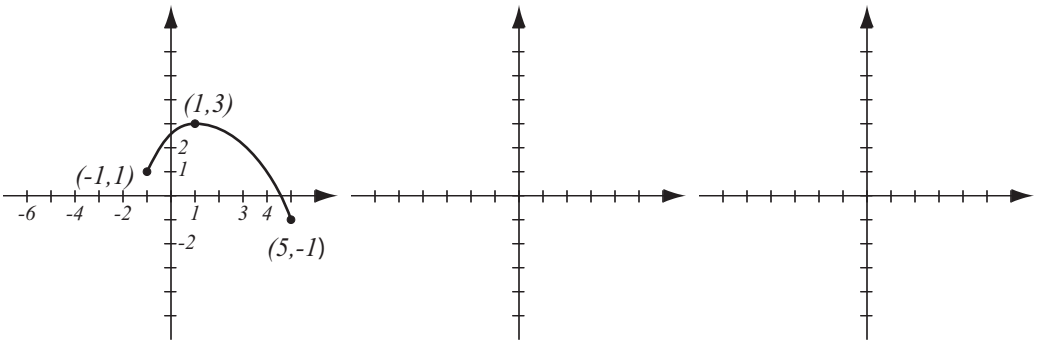
4. (6pts) Solve and write the solution in interval notation.

$2x - 7 < 3$ or $3x - 5 > 20$

5. (4pts) Find the domain of the function $f(x) = \dfrac{1}{x^2 + 4x - 21}$ and write it in interval notation.

6. (6pts) Let $f(x) = \dfrac{4x}{x-3}$. Find the formula for f^{-1}.

7. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $2f(x+3)$ and $f(-x)+2$ and label all the relevant points.



8. (12pts) The quadratic function $f(x) = -x^2 - 4x + 5$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

9. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_2 \frac{32\sqrt[5]{y^8}}{x^9} =$$

10. (5pts) Write as a single logarithm. Simplify if possible.

$$3\log(x^{-2}y^4) - \log(x^3y^{-5}) =$$

11. (20pts) The polynomial $P(x) = x^3 - 25x$ is given (answer with 6 decimals accuracy).
a) What is the end behavior of the polynomial?
b) Factor the polynomial to find all the zeros and their multiplicities. Find the y-intercept.
c) Determine algebraically whether the function is odd, even, or neither.
d) Use the graphing calculator along with a) and b) to sketch the graph of P (yes, on paper!).
e) Verify your conclusion from c) by stating symmetry.
f) Find all the turning points (i.e., local maxima and minima).

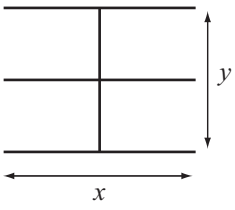
Solve the equations.

12. (8pts) $\dfrac{2x}{x+4} + \dfrac{10x-8}{x^2+2x-8} = \dfrac{x}{x-2}$

13. (6pts) $3^{x-1} = 9^{x-2}$

14. (14pts) Shepherd Billy looked away from his cow just when it started trotting away at 2 meters per second. Having realized what is happening 10 seconds later, he starts to chase the cow, running at 5 meters per second.
a) How long does Billy run until he catches up with the cow?
b) How far does he run until that moment?

15. (14pts) A local businesswoman is building a repair shop with 4 bays, as in the picture. She has enough money to build 220 feet of walls, and her goal is to maximize the total area of the shop.
a) Express the total area of the shop as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the shop that has the biggest possible total area, and what is the biggest possible total area?



16. (12pts) The population of Breedington was 11,000 in 2012 and 13,000 in 2015. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2012. Graph it on paper.
b) Find the predicted population in the year 2017.

Bonus (10pts) Verify, by plugging in and doing the algebra, that all of the numbers $-2i$, $\sqrt{3} + i$ and $-\sqrt{3} + i$ are solutions of the equation $z^3 = 8i$. One is easy and two are not so easy. (This is an illustration of a general fact: every equation $z^3 = b$ has three solutions among complex numbers.)