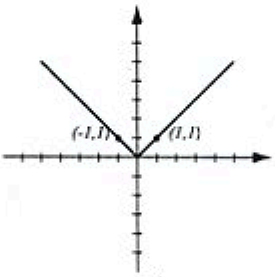
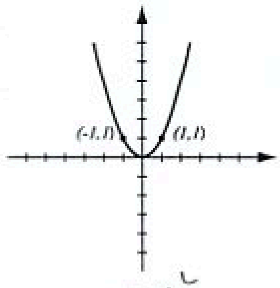
College Algebra — Final Exam **MAT 140, Spring 2018 — D. Ivanšić**	**Name:** Saul Ocean *Show all your work!*

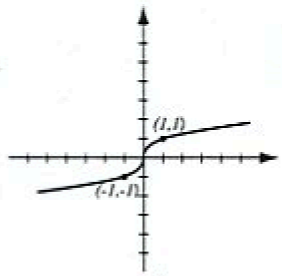
1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



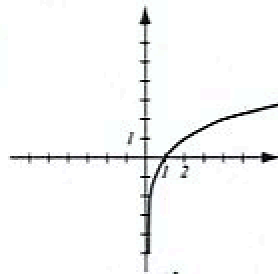
$y = |x|$



$y = x^2$

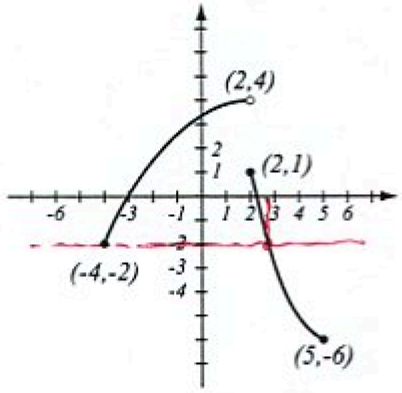


$y = \sqrt[3]{x}$



$y = \log_a x, \quad a > 1$

2. (8pts) Use the graph of the function f at right to answer the following questions.



a) Find: $f(-3) =$ 0 $\quad f(2) =$ 1

b) What is the domain of f? $[-4, 5]$

c) What is the range of f? $[-6, 4]$

d) What are the solutions of the equation $f(x) = -2$?

$x = -4, 2.7$

3. (12pts) a) Write the equation of the line whose y-intercept is 2 and has slope 3.
b) Write the equation of the line through points $(-1, 3)$ and $(2, 2)$.
c) Are the two lines perpendicular?
d) Draw both lines.

a) $y = 3x + 2$

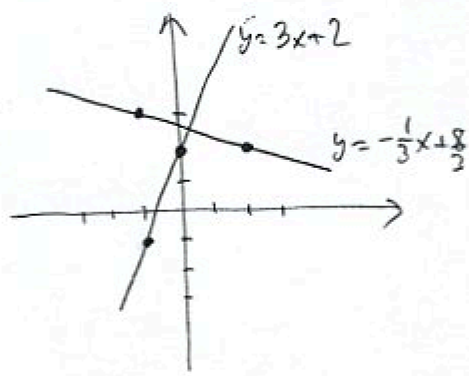
b) $m = \frac{2-3}{2-(-1)} = \frac{-1}{3} = -\frac{1}{3}$

$y - 3 = -\frac{1}{3}(x - (-1))$

$y = -\frac{1}{3}x - \frac{1}{3} + 3$

$y = -\frac{1}{3}x + \frac{8}{3}$

c) Yes, their slopes 3 and $-\frac{1}{3}$ are opposite reciprocal

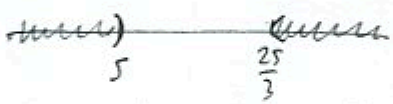


4. (6pts) Solve and write the solution in interval notation.

$2x - 7 < 3$ or $3x - 5 > 20$

$2x < 10 \qquad 3x > 25$

$x < 5 \quad$ or $\quad x > \frac{25}{3}$



$(-\infty, 5) \cup (\frac{25}{3}, \infty)$

5. (4pts) Find the domain of the function $f(x) = \dfrac{1}{x^2+4x-21}$ and write it in interval notation.

Can't have $x^2+4x-21=0$

$(x+7)(x-3)=0$

$x=-7,3$

$(-\infty,-7)\cup(3,\infty)$

$-7 \qquad 3$

6. (6pts) Let $f(x) = \dfrac{4x}{x-3}$. Find the formula for f^{-1}.

$y = \dfrac{4x}{x-3}$

$y(x-3) = 4x$

$yx - 3y = 4x$

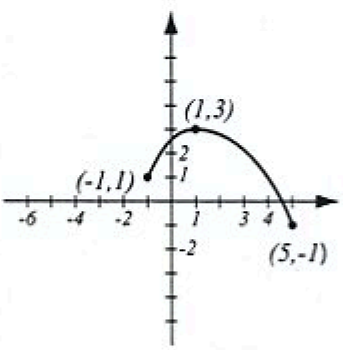
$yx - 4x = 3y$

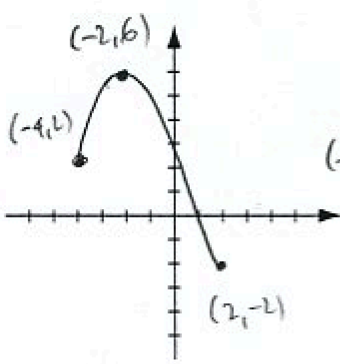
$x(y-4) = 3y$

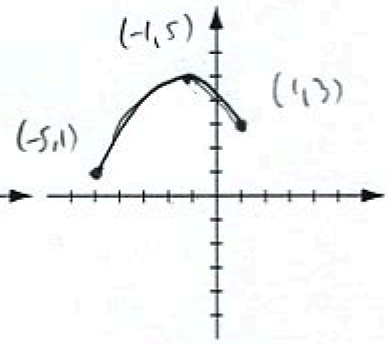
$x = \dfrac{3y}{y-4}$

$f^{-1}(y) = \dfrac{3y}{y-4}$

7. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $2f(x+3)$ and $f(-x)+2$ and label all the relevant points.







Shift left 3
stretch vertically,
factor = 2

Reflect in y-axis,
shift up 2

8. (12pts) The quadratic function $f(x) = -x^2 - 4x + 5$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

a) $-x^2-4x+5=0$

$x^2+4x-5=0$

$(x+5)(x-1)=0$

$x=-5,1$

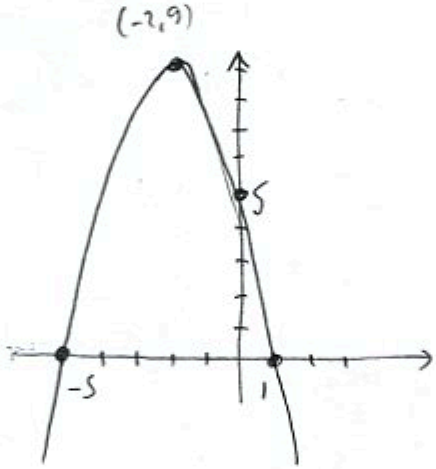
y-int: $f(0)=5$

b) $h=-\frac{-4}{2\cdot(-1)}=-2$

$k=f(-2)=-(-2)^2-4(-2)+5$

$=-4+8+5$

$=9$



9. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_2 \frac{32\sqrt[5]{y^8}}{x^9} = \log_2 32+\log_2 y^{\frac{8}{5}}-\log_2 x^9$$

$$=5+\frac{8}{5}\log_2 y-9\log_2 x$$

10. (5pts) Write as a single logarithm. Simplify if possible.

$$3\log(x^{-2}y^4)-\log(x^3y^{-5}) = \log(x^{-2}y^4)^3-\log(x^3y^{-5})$$

$$=\log\frac{(x^{-2}y^4)^3}{x^3y^{-5}}=\log\frac{x^{-6}y^{12}}{x^3y^{-5}}=\log(x^{-9}y^{17})=\log\frac{y^{17}}{x^{-5}}$$

11. (20pts) The polynomial $P(x) = x^3 - 25x$ is given (answer with 6 decimals accuracy).
a) What is the end behavior of the polynomial?
b) Factor the polynomial to find all the zeros and their multiplicities. Find the y-intercept.
c) Determine algebraically whether the function is odd, even, or neither.
d) Use the graphing calculator along with a) and b) to sketch the graph of P (yes, on paper!).
e) Verify your conclusion from c) by stating symmetry.
f) Find all the turning points (i.e., local maxima and minima).

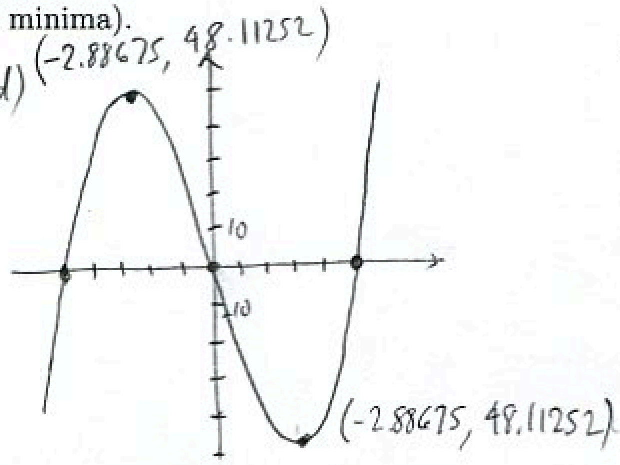
a) Behaves like x^3

b) $x^3 - 25x = x(x^2 - 25) = x(x-5)(x+5)$

zero	0	-5	5
mult.	1	1	1

c) $P(-x) = (-x)^3 - 25(-x)$
$= -x^3 + 25x = -P(x)$
so odd

d)



e) It is symmetric wrt origin

f) $f(-2.88675) = 48.11252$ is a local max,
$f(2.88675) = -48.11252$ is a local min.

Solve the equations.

12. (8pts) $\dfrac{2x}{x+4} + \dfrac{10x-8}{\underset{(x+4)(x-2)}{x^2+2x-8}} = \dfrac{x}{x-2}$ $\Big| \cdot (x+4)(x-2)$

$\dfrac{2x}{\cancel{x+4}} \cdot \cancel{(x+4)}(x-2) + \dfrac{10x-8}{\cancel{(x+4)}\cancel{(x-2)}} \cancel{(x+4)}\cancel{(x-2)} = \dfrac{x}{\cancel{x-2}} \cdot (x+4)\cancel{(x-2)}$

$2x(x-2) + 10x - 8 = x(x+4)$

$2x^2 - 4x + 10x - 8 = x^2 + 4x$ $\Big| -x^2 - 4x$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4, 2$ both give a 0 in the denom. so are not solutions

$\boxed{\text{No sol.}}$

13. (6pts) $3^{x-1} = 9^{x-2}$

$3^{x-1} = (3^2)^{x-2}$

$3^{x-1} = 3^{2x-4}$

$x - 1 = 2x - 4$ $\Big| -x + 4$

$\boxed{3 = x}$

14. (14pts) Shepherd Billy looked away from his cow just when it started trotting away at 2 meters per second. Having realized what is happening 10 seconds later, he starts to chase the cow, running at 5 meters per second.

a) How long does Billy run until he catches up with the cow?

b) How far does he run until that moment?

Cow $d, 2, t$

Billy $d, 5, t-10$

a) $16\frac{2}{3}$ seconds

b) $33\frac{1}{3}$ meters

Same $\begin{cases} d = 2t \\ d = 5(t-10) \end{cases}$ $t = \frac{50}{3} = 16\frac{2}{3}$ seconds

$2t = 5(t-10)$ $d = 2 \cdot \frac{50}{3} = \frac{100}{3} = 33\frac{1}{3}$ meters

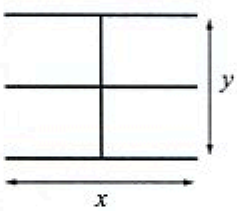
$2t = 5t - 50$

$50 = 3t$

15. (14pts) A local businesswoman is building a repair shop with 4 bays, as in the picture. She has enough money to build 220 feet of walls, and her goal is to maximize the total area of the shop.

a) Express the total area of the shop as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the shop that has the biggest possible total area, and what is the biggest possible total area?

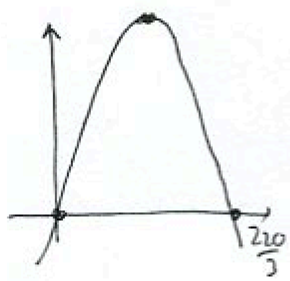


a) $A = xy = x(220-3x) = -3x^2 + 220x$

$3x + y = 220$

$y = 220 - 3x$

b)



$h = -\frac{b}{2a} = -\frac{220}{-6} = \frac{110}{3}$

$k = \frac{110}{3} \cdot \left(220 - 3 \cdot \frac{110}{3}\right) = \frac{110}{3} \cdot 110$

$= \frac{12100}{3}$

Domain:
Must have $x \geq 0$
$y \geq 0$

$220 - 3x \geq 0$

$3x \leq 220$

$x \leq \frac{220}{3}$

Dimensions of shop: $\frac{110}{3} \times 110$

Maximal area: $\frac{12100}{3}$

Domain: $\left[0, \frac{220}{3}\right]$

$t=0$ $t=3$

16. (12pts) The population of Breedington was 11,000 in 2012 and 13,000 in 2015. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2012. Graph it on paper.
b) Find the predicted population in the year 2017. 5 $t=5$

a) $P(t) = 11e^{kt}$

$P(3) = 13$

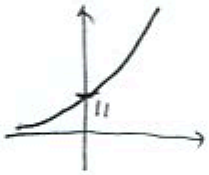
$11e^{k\cdot 3} = 13$

$e^{k\cdot 3} = \frac{13}{11}$ | ln

$\ln e^{k\cdot 3} = \ln \frac{13}{11}$

$3k = \ln \frac{13}{11}$

$k = \frac{\ln \frac{13}{11}}{3} = 0.0556847$



b) Need $P(5) = 11e^{0.0556847\cdot 5}$

$= 14.531501$

About 14,532 people

Bonus (10pts) Verify, by plugging in and doing the algebra, that all of the numbers $-2i$, $\sqrt{3}+i$ and $-\sqrt{3}+i$ are solutions of the equation $z^3 = 8i$. One is easy and two are not so easy. (This is an illustration of a general fact: every equation $z^3 = b$ has three solutions among complex numbers.)

$(-2i)^3 = (-2)^3 i^3 = -8\cdot(-i) = 8i$

$(\sqrt{3}+i)^3 = (\sqrt{3}+i)^2(\sqrt{3}+i) = (\underbrace{\sqrt{3}^2}_{3} + 2\sqrt{3}\cdot i + \underbrace{i^2}_{-1})(\sqrt{3}+i) = (2+2\sqrt{3}i)(\sqrt{3}+i)$

$= 2\sqrt{3} + 2i + 2\sqrt{3}^2 i + 2\sqrt{3}i^2 = 2\sqrt{3} + 2i + 6i - 2\sqrt{3} = 8i$

$(-\sqrt{3}+i)^3 = (i-\sqrt{3})^2(i-\sqrt{3}) = (\underbrace{i^2}_{-1} - 2i\sqrt{3} + \underbrace{\sqrt{3}^2}_{3})(i-\sqrt{3}) = (2-2i\sqrt{3})(i-\sqrt{3})$

$= 2i - 2\sqrt{3} - 2\sqrt{3}i^2 + 2i\sqrt{3}^2 = 2i - 2\sqrt{3} + 2\sqrt{3} + 6i = 8i$