College Algebra — Exam 4
MAT 140, Spring 2018 — D. Ivanšić

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Show all your work!

1. (8pts) Evaluate without using the calculator:

$\log_3 81 =$ 4 $\qquad$ $\log_2 \frac{1}{16} =$ -4 $\qquad$ $\log_a \sqrt{a^7} =$ $\frac{7}{2}$ $\qquad$ $\log_{b^3} b^{12} =$ 4

$3^? = 81$ $\qquad$ $2^? = \frac{1}{16} = \frac{1}{2^4} = 2^{-4}$ $\qquad$ $a^? = \sqrt{a^7} = a^{\frac{7}{2}}$ $\qquad$ $(b^3)^? = b^{12}$ $\quad$ $3 \cdot ? = 12$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_7 0.56$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_7 0{,}56 = \frac{\ln 0{,}56}{\ln 7} = -0{,}297968$

3. (5pts) If $\log_a 5 = u$ and $\log_a 4 = v$, express in terms of u and v:

$\log_a 20 = \log_a (5 \cdot 4)$
$= \log_a 5 + \log_a 4$
$= u + v$

$\log_a \frac{5}{16} = \log_a 5 - \log_a 16$
$= \log_a 5 - \log_a 4^2$
$= \log_a 5 - 2\log_a 4$
$= u - 2v$

4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_7 \frac{y^4}{49\sqrt[3]{x^4}} = \log_7 y^4 - \log_7 49 - \log_7 x^{\frac{4}{3}}$
$= 4\log_7 y - 2 - \frac{4}{3}\log_7 x$

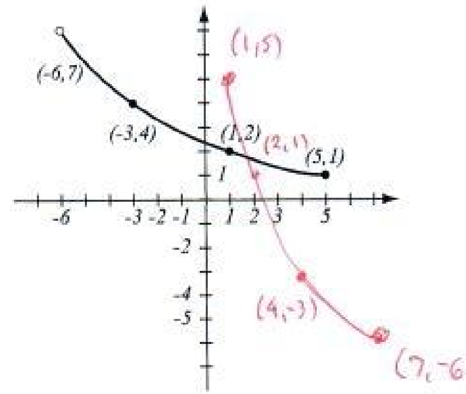
5. (6pts) Write as a single logarithm. Simplify if possible.

$3\log(x^2y^{-3}) - 2\log(x^4y) = \log (x^2y^{-3})^3 - \log (x^4y)^2$
$= \log \frac{(x^2y^{-3})^3}{(x^4y)^2} = \log \frac{x^6y^{-9}}{x^8y^2} = \log x^{-2}y^{-11} = \log \frac{1}{x^2y^{11}}$

6. (4pts) Simplify.

$\ln e^{3x-4} =$ $3x - 4$ $\qquad$ $6^{\log_6 \sqrt{2}} =$ $\sqrt{2}$

7. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points, and showing any asymptotes.



a) yes - it passes the horizontal line test
b) Reflect graph in line y=x
(swap coordinates of points

8. (9pts) Let $f(x) = \dfrac{x-3}{4x}$.
a) Find the formula for f^{-1}.
b) Find the range of f.

$y = \dfrac{x-3}{4x}$

$4xy = x-3$

$4xy - x = -3$

$x(4y-1) = -3$

$x = -\dfrac{3}{4y-1} = \dfrac{3}{1-4y}$

$f^{-1}(y) = \dfrac{3}{1-4y}$

b) Range of f = domain of f^{-1}

Can't have: $1-4y=0$

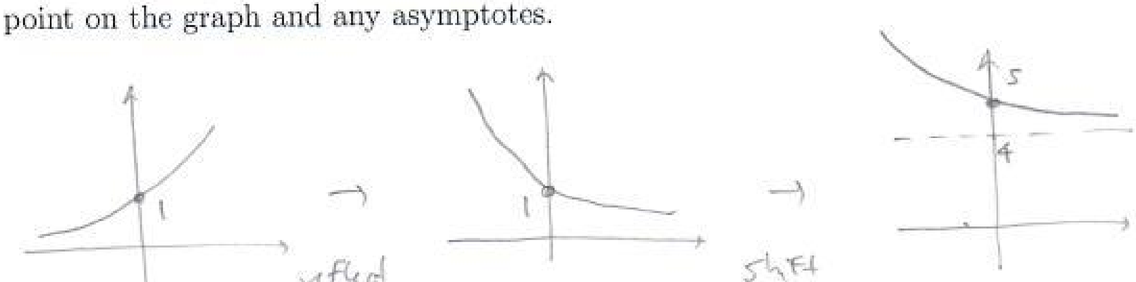
$1 = 4y$

$y = \frac{1}{4}$

$\frac{1}{4}$

Range of f: $(-\infty, \frac{1}{4}) \cup (\frac{1}{4}, \infty)$

9. (6pts) Using transformations, draw the graph of $f(x) = 4 + e^{-x}$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.



$y=e^x$ → reflect in y-axis → $y=e^{-x}$ → shift up 4 → $y=4+e^{-x}$

10. (3pts) Find the domain of the function $f(x) = \log_5(4x+9)$ and write it in interval notation.

Must have

$4x+9>0$

$4x>-9$

$x>-\frac{9}{4}$

$(-\frac{9}{4}, \infty)$

11. (9pts) What is better: an account bearing 5.1% compounded monthly, or an account bearing 5.2% compounded quarterly? Find out by comparing \$100 deposits placed for a year.

$A = P(1+\frac{r}{n})^{nt}$

$A = 100(1+\frac{0.051}{12})^{12\cdot 1}$ or $A = 100(1+\frac{0.052}{4})^{4}$

$= 105.22$ $\qquad = 105.30$

5.2% comp. quarterly is slightly better.

Solve the equations.

12. (6pts) $2^{2x-1} = 8^{x-3}$

$2^{2x-1} = (2^3)^{x-3}$

$2^{2x-1} = 2^{3x-9}$

$2x-1 = 3x-9 \quad | -2x+9$

$8 = x$

13. (8pts) $5^{x+3} = 9^{2x}$ $\quad | \ln$

$\ln 5^{x+3} = \ln 9^{2x}$

$(x+3)\ln 5 = 2x \ln 9$

$x \ln 5 + 3\ln 5 = 2\ln 9 x$

$\ln 5 x - 2\ln 9 x = -3\ln 5$

$x(\ln 5 - 2\ln 9) = -3\ln 5$

$x = \frac{-3\ln 5}{\ln 5 - 2\ln 9} = \frac{3\ln 5}{2\ln 9 - \ln 5} = 1.733678$

14. (8pts) $\log_2(2x-3) - \log_2(x-7) = 2$

$\log_2 \frac{2x-3}{x-7} = 2$

$2^{\log_2 \frac{2x-3}{x-7}} = 2^2$

$\frac{2x-3}{x-7} = 4$

$2x-3 = 4(x-7)$

$2x-3 = 4x-28$

$25 = 2x$

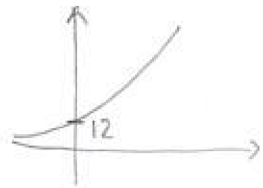
$x = \frac{25}{2}$ ← does not give negatives inside of logs

$2\cdot\frac{25}{2} - 3 = 22 > 0$

$\frac{25}{2} - 7 = \frac{11}{2} > 0$

15. (12pts) The population of Breedington was 12,000 in 2011 and 14,000 in 2015. Assume that it has grown according to the formula $P(t) = P_0e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2011. Graph it on paper.
b) Find the predicted population in the year 2020.

a) $P(t) = 12e^{kt}$ (in thousands)



$P(4) = 14$

$12e^{k4} = 14 \quad | \div 12$

$e^{4k} = \frac{14}{12} = \frac{7}{6} \quad | \ln$

$\ln e^{4k} = \ln \frac{7}{6}$

$4k = \ln \frac{7}{6}$

$k = \frac{\ln \frac{7}{6}}{4} = 0.0385377$

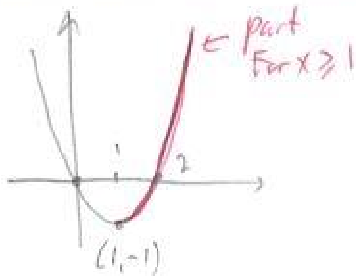
b) $P(9) = 12e^{0.0385377 \cdot 9}$

$= 16.975068$

In 2020 population is predicted to be about 16,975.

Bonus (10pts) Let $f(x) = x^2 - 2x$, with domain $x \geq 1$.
a) Graph the function (sketch on paper!). Explain why it is one-to-one.
b) Find the formula for $f^{-1}(x)$. (Once you set it up, solving for x involves doing a quadratic equation, which you solve using the quadratic formula.)

a)



Red part passes horizontal line test.

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, 2$ x-int

b) $y = x^2 - 2x$

$x^2 - 2x - y = 0$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-y)}}{2 \cdot 1}$

$= \frac{2 \pm \sqrt{4 + 4y}}{2} = \frac{2 \pm \sqrt{4(1+y)}}{2}$

$= \frac{2 \pm 2\sqrt{1+y}}{2} = 1 \pm \sqrt{1+y}$

$x = 1 \pm \sqrt{1+y}$

Since $x \geq 1$, we take the + solution

$f^{-1}(y) = 1 + \sqrt{1+y}$