College Algebra — Exam 3
MAT 140, Spring 2018 — D. Ivanšić

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Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (5pts) $i(3-i)+3(1-2i) = 3i-\underbrace{i^2}_{-1}+3-6i = 4-3i$

2. (5pts) $\dfrac{2-i}{1+3i} = \dfrac{2-i}{1+3i}\cdot\dfrac{1-3i}{1-3i} = \dfrac{2-6i-i+3i^2}{1^2-(3i)^2} = \dfrac{-1-7i}{1+9\underbrace{i^2}_{=-1}} = \dfrac{-1-7i}{10}$

3. (4pts) Simplify and justify your answer.

$i^{218} = i^{216}\cdot i^2 = (\underbrace{i^4}_{=1})^{54}\cdot i^2 = i^2 = -1$

4. (6pts) Solve the equation by completing the square. 16.2

$x^2 - 14x + 17 = 0 \quad |+7^2-17$

$x^2-2\cdot x\cdot 7+7^2 = -17+7^2$

$(x-7)^2 = 32$

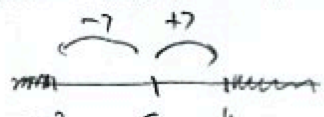
$x-7 = \pm\sqrt{32}$

$x = 7\pm 4\sqrt{2}$

5. (6pts) Solve the inequality. Write the solution in interval form.

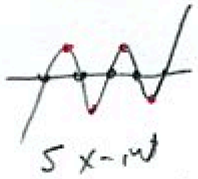
$|x-4| \geq 7$

distance from x to $4 \geq 7$ $\quad (-\infty,-3]\cup[11,\infty)$



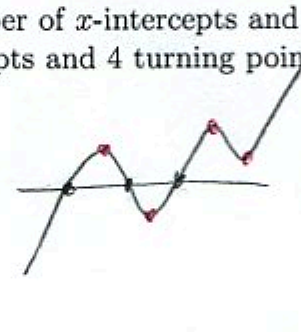
6. (6pts) Let $P(x)$ be a polynomial of degree 5.
a) What is the maximal number of x-intercepts that $P(x)$ can have? The maximal number of turning points?
b) Draw a graph of P that has the maximal number of x-intercepts and turning points.
c) Draw a graph of P that has exactly 3 x-intercepts and 4 turning points.

a) 5 and 4 $\quad$ b)



5 x-int
4 turning pts

c)



3 x-int
4 turning points

7. (12pts) The quadratic function $f(x) = x^2 - 2x + 5$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

a) y-int:

$f(0) = 5$

x-int:

$x^2 - 2x + 5 = 0$ 4-20

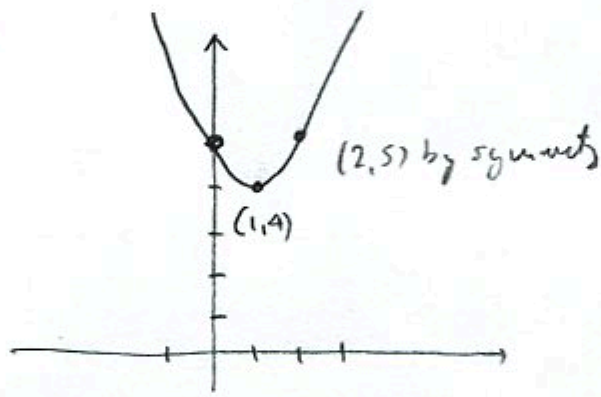
$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1}$

$= \dfrac{2 \pm \sqrt{-16}}{2}$ not real, so no x-int.

b) $h = -\dfrac{b}{2a} = -\dfrac{-2}{2 \cdot 1} = 1$

$k = f(1) = 1 - 2 + 5 = 4$

c)



Solve the equations:

8. (8pts) $\dfrac{x}{x+1} + \dfrac{16}{x^2 - 6x - 7} = \dfrac{2}{x-7}$ $\Big| \cdot (x+1)(x-7)$

$(x+1)(x-7)$

$\dfrac{x}{\cancel{x+1}}(\cancel{x+1})(x-7) + \dfrac{16}{\cancel{(x+1)}\cancel{(x-7)}}\cancel{(x+1)}\cancel{(x-7)} = \dfrac{2}{\cancel{x-7}}(x+1)\cancel{(x-7)}$

$x(x-7) + 16 = 2(x+1)$

$x^2 - 7x + 16 = 2x + 2$ $\quad | - 2x + 2$

$x^2 - 9x + 14 = 0$

$(x-2)(x-7) = 0$

$x = \boxed{2}, 7$

only sol.

gives 0 in a denom. in the original eq.

9. (8pts) $1 - \sqrt{19 + 6x} = x + 3$ $\quad | -1$

$-\sqrt{19+6x} = x + 2$ $\quad | ^2$

$\left(-\sqrt{19+6x}\right)^2 = (x+2)^2$

$19 + 6x = x^2 + 4x + 4$ $\quad | -6x - 19$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x = 5$ or $\boxed{x = -3}$

check: $1 - \sqrt{19+30} = 5+3$ $\quad$ $1 - \sqrt{19-18} = -3+3$

$1 - 7 = 8$ $\quad$ $1 - 1 = 0$

no $\quad$ yes

10. (14pts) The polynomial $f(x) = (x-5)(x+2)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

a) like $x \cdot x^2 = x^3$

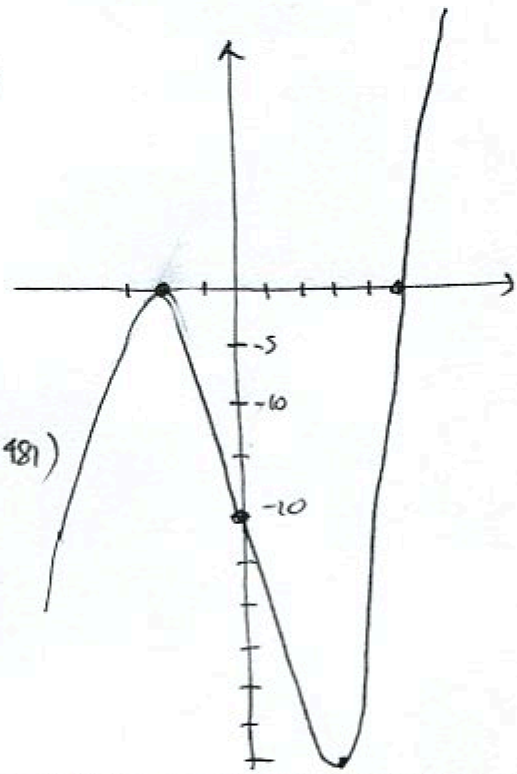
b)

zero	5	-2
mult.	1	2

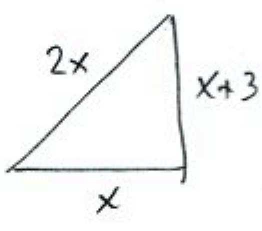
y-int, $f(0) = -5 \cdot 2^2 = -20$

d) Turning points; $(-2, 0)$
$(2.666667, -50.81481)$

c)



11. (12pts) In a right triangle, the hypothenuse is twice the length of the shorter side, and the longer side is 3 centimeters longer than the shorter side. What is the length of the shorter side in this right triangle?



$$x^2 + (x+3)^2 = (2x)^2$$
$$x^2 + x^2 + 6x + 9 = 4x^2$$
$$-2x^2 + 6x + 9 = 0 \quad | \cdot (-1)$$
$$2x^2 - 6x - 9 = 0$$

$$x = \frac{-(-6) \pm \sqrt{\overbrace{(-6)^2 - 4 \cdot 2(-9)}^{36+72}}}{2 \cdot 2} = \frac{6 \pm \sqrt{\overbrace{108}^{36 \cdot 3}}}{4} = \frac{6 \pm 6\sqrt{3}}{4}$$

$$= \frac{2(3 \pm 3\sqrt{3})}{4} = \frac{3 \pm 3\sqrt{3}}{2} = 4.098076, -1.098076$$

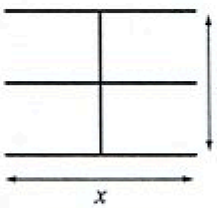
the only solution.

$x \geq 0$ so not a sol

12. (14pts) A local businesswoman is building a repair shop with 4 bays, as in the picture. She has enough money to build 300 feet of walls, and her goal is to maximize the total area of the shop.

a) Express the total area of the shop as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the shop that has the biggest possible total area, and what is the biggest possible total area?



a) length of walls $= 300$

$3x + y = 300$

$y = 300 - 3x$

$A = x \cdot y = x(300 - 3x)$

$= -3x^2 + 300x$

Domain: Must have:

$x \geq 0$

$y \geq 0$

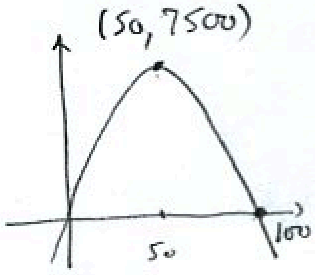
$300 - 3x \geq 0$

$3x \leq 300$

$x \leq 100$

$[0, 100]$

b)



$h = -\frac{b}{2a} = -\frac{300}{2 \cdot (-3)} = 50$

$A(50) = -3 \cdot 50^2 + 300 \cdot 50$

$= 7500 \text{ ft}^2$

$y = 300 - 3 \cdot 50$

Dimensions of shop: 50×150

Maximal area : 7500 ft^2

Bonus. (10pts) Verify, by plugging in and doing the algebra, that all of the numbers $-2i$, $\sqrt{3} + i$ and $-\sqrt{3} + i$ are solutions of the equation $z^3 = 8i$. One is easy and two are not so easy. (This is an illustration of a general fact: every equation $z^3 = b$ has three solutions among complex numbers.)

$(-2i)^3 = (-2)^3 i^3 = -8 \cdot \underbrace{i \cdot i}_{=-1} \cdot i = 8i$

$(\sqrt{3}+i)^3 = (\sqrt{3}+i)^2(\sqrt{3}+i) = (\sqrt{3}^2 + 2\sqrt{3}i + \underbrace{i^2}_{=-1})(\sqrt{3}+i) = (2 + 2\sqrt{3}i)(\sqrt{3}+i)$

$= \cancel{2\sqrt{3}} + 2i + 2\sqrt{3}^2 i + \cancel{2\sqrt{3}}\underbrace{i^2}_{=-1} = 2i + 6i = 8i$

$(-\sqrt{3}+i)^3 = (i - \sqrt{3})^3 = (i-\sqrt{3})^2(i-\sqrt{3}) = (\underbrace{i^2}_{=-1} - 2i\sqrt{3} + \sqrt{3}^2)(i - \sqrt{3})$

$= (2 - 2i\sqrt{3})(i - \sqrt{3}) = 2i - \cancel{2\sqrt{3}} - \cancel{2\sqrt{3}}\underbrace{i^2}_{=-1} + 2i\sqrt{3}^2$

$= 2i + 6i = 8i$