College Algebra — Exam 2
MAT 140, Spring 2018 — D. Ivanšić

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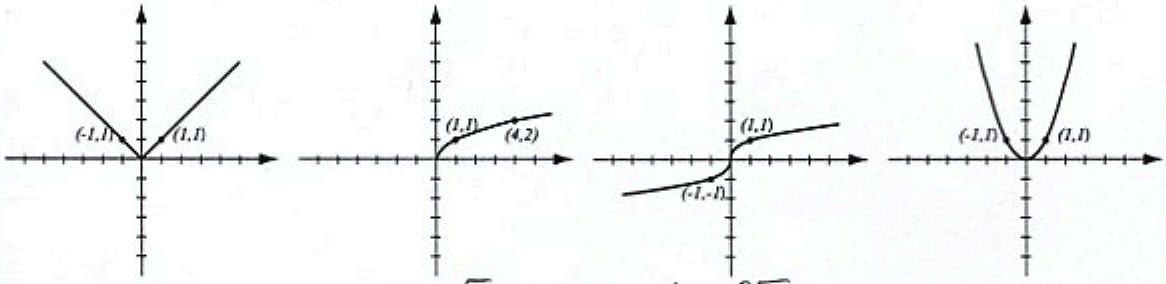
Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y = |x|$ $\qquad$ $y = \sqrt{x}$ $\qquad$ $y = \sqrt[3]{x}$ $\qquad$ $y = x^2$

2. (21pts) Let $f(x) = \dfrac{3x-2}{x+1}$, $g(x) = \dfrac{1}{x-4}$.

Find the following (simplify where possible):

$(f+g)(1) = f(1) + g(1)$

$= \dfrac{3-2}{1+1} + \dfrac{1}{1-4} = \dfrac{1}{2} + \dfrac{1}{-3} = \dfrac{1}{6}$

$(fg)(-2) = f(-2) \cdot g(-2)$

$= \dfrac{-6-2}{-2+1} \cdot \dfrac{1}{-2-4} = \dfrac{-\cancel{8}\,4}{-1} \cdot \dfrac{1}{-\cancel{6}\,3} = -\dfrac{4}{3}$

$\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{\frac{3x-2}{x+1}}{\frac{1}{x-4}}$

$= \dfrac{3x-2}{x+1} \cdot \dfrac{x-4}{1} = \dfrac{(3x-2)(x-4)}{x+1}$

$(f \circ g)(5) = f(g(5))$

$= f\left(\dfrac{1}{5-4}\right) = f(1) = \dfrac{3-2}{1+1} = \dfrac{1}{2}$

$(g \circ f)(x) = g(f(x)) = g\left(\dfrac{3x-2}{x+1}\right) = \dfrac{1}{\frac{3x-2}{x+1} - 4} = \dfrac{1}{\frac{3x-2-4(x+1)}{x+1}} = \dfrac{1}{\frac{-x-6}{x+1}}$

$= \dfrac{x+1}{-x-6} = -\dfrac{x+1}{x+6}$

The domain of $f - g$ in interval notation

domain f: can't have: $x+1=0$

$x = -1$

domain g: can't have $x-4=0$

$x = 4$

dom f

dom g

dom f-g

-1 $\qquad$ 4

= overlap

$(-\infty,-1) \cup (-1,4) \cup (4,\infty)$

3. (6pts) Consider the function $h(x) = 5(x+1)^2$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

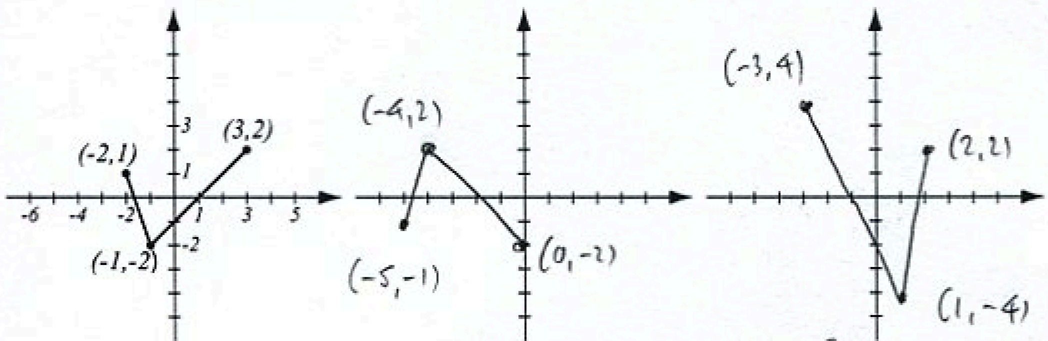
$g(x) = x+1 \qquad g(x) = (x+1)^2$

$f(x) = 5x^2 \qquad f(x) = 5x$

4. (6pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = \sqrt{x}$, shifted right 4 units
b) shape of $y = x^2$, stretched horizontally by factor 3, then shifted down 5 units.

a) $y = \sqrt{x-4}$ b) $y = x^2 \to \left(\frac{1}{3}x\right)^2 \longrightarrow \left(\frac{1}{3}x\right)^2 - 5$

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x+3)$ and $2f(-x)$ and label all the relevant points.



shift left 3,
reflect in x-axis

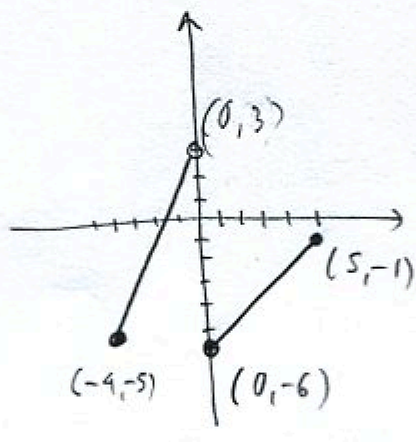
reflect in y-axis
stretch vertically, factor = 4

6. (8pts) Sketch the graph of the piecewise-defined function:

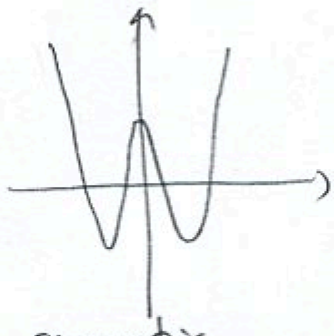
$$f(x) = \begin{cases} 2x+3, & \text{if } -4 \le x < 0 \\ x-6, & \text{if } 0 \le x \le 5 \end{cases}$$

x	2x+3
-4	-5
0	3

x	x-6
0	-6
5	-1



7. (7pts) For the function $f(x) = x^4 - 6x^2 + 5$:
a) Determine algebraically whether it is odd, even, or neither.
b) Use the calculator to sketch its graph here and verify your conclusion by stating symmetry.



symmetric
wrt y-axis

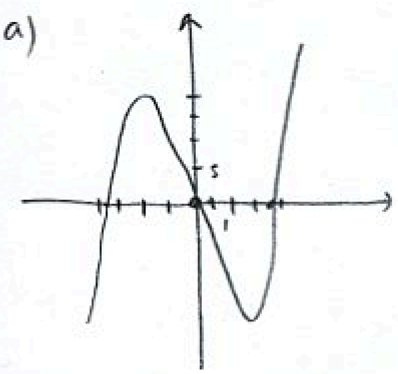
a) $f(-x) = (-x)^4 - 6(-x)^2 + 5$

$= x^4 - 6x^2 + 5 = f(x)$

so even

8. (20pts) Let $f(x) = x^3 - 12x$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.
b) Determine algebraically whether the function is odd, even, or neither.
c) Verify your conclusion from b) by stating symmetry.
d) Find the local maxima and minima for this function.
e) State the intervals where the function is increasing and where it is decreasing.

a)



d) $f(-2) = 16$ is a local max

$f(2) = -16$ is a local min

e) increasing on $(-\infty, -2)$ and $(2, \infty)$

decreasing on $(-2, 2)$

b) $f(-x) = (-x)^3 - 12(-x)$

$= -x^3 + 12x = -f(x)$

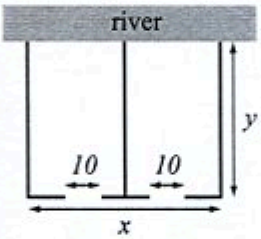
so odd

c) graph is symmetric
wrt. origin

9. (14pts) Farmer Joe is fencing a rectangular field next to a river, dividing it in two sections and leaving a 10-foot opening in each section. The side along the river does not require fencing. The field must have area 6,000 square feet and Joe's goal is to minimize the total length of the fence.

a) Express the total fence length as a function of the length of one of the sides x. What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the field that has the smallest total fence length and what is the minimal fence length?



a) $\ell = x - 10 - 10 + 3y$

$= x + 3y - 20$

$= x + 3 \cdot \frac{6000}{x} - 20$

$= x + \frac{18000}{x} - 20$

$xy = 6000$

$y = \frac{6000}{x}$

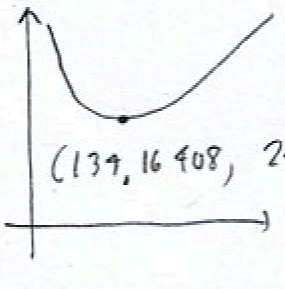
Domain: must have: $x \geq 20$

Domain: $y > 0$

$[20, \infty)$ $\quad \frac{6000}{x} > 0$

which holds

b)



dimensions are x $\quad y = \frac{6000}{x}$

$134.16408 \times 44.72136$ feet

minimal fence length

$= 248.32816$ feet

Bonus. (10pts) Consider all lines through point $(2, 1)$ with **negative** slope.

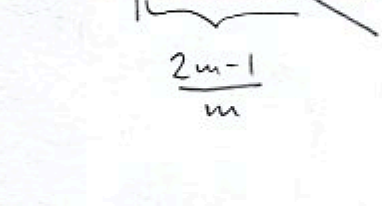
a) Draw one such line. With the axes, it forms a triangle in the first quadrant whose area depends on the slope m.

b) Write the equation of a line through $(2, 1)$ with slope m (the equation will have m in it).

c) Determine the x- and y-intercepts of the line in b). They will depend on m.

d) Using c), write the expression for the area of the triangle described in a). It will depend only on m.

a)



b) $y - 1 = m(x - 2)$

$y = mx - 2m + 1$

c) y-int is $-2m+1$

x-int: $0 = mx - 2m + 1$

$2m - 1 = mx$

$x = \frac{2m-1}{m}$

d) Area $= \frac{\text{base} \cdot \text{height}}{2}$

$= \frac{2m-1}{m} \cdot (-2m+1) \cdot \frac{1}{2}$

$= -\frac{(2m-1)^2}{m} \cdot \frac{1}{2} = \frac{(2m-1)^2}{-2m}$