1. (8pts) Use the graph of the function f at right to answer the following questions.
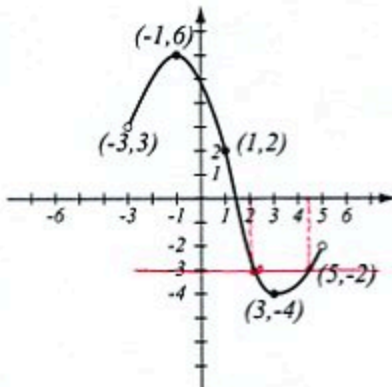
a) Find: $f(3) =$ -4 $f(-3) =$ not defined

b) What is the domain of f? $(-3, 5)$

c) What is the range of f? $[-4, 6]$

d) What are the solutions of the equation $f(x) = -3$? $x = 2, 4.5$

(-1,6)
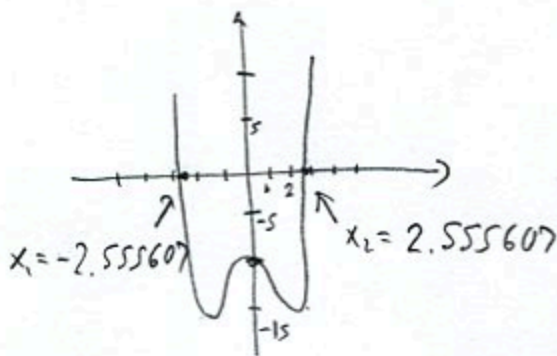(-3,3)   (1,2)
(3,-4)   (5,-2)

2. (10pts) Use your calculator to accurately sketch the graph of $y = x^4 - 5x^2 - 10$.

a) Draw the graph on paper and indicate units on the axes.

b) Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).

y-int: $x=0$, $y = -10$

x-int: on graph

$x_1 = -2.555607$ $x_2 = 2.555607$

3. (5pts) Write the equation of the line that passes through points $(2, -3)$ and $(-4, 9)$.

$m = \dfrac{9-(-3)}{-4-2} = \dfrac{12}{-6} = -2$

$y - (-3) = -2(x-2)$

$y + 3 = -2x + 4$

$y = -2x + 1$

4. (10pts) Find the equation of the line (in form $y = mx + b$) that is parallel to the line $3x - 2y = 8$ and has x-intercept 5. Draw both lines.

$3x - 2y = 8$

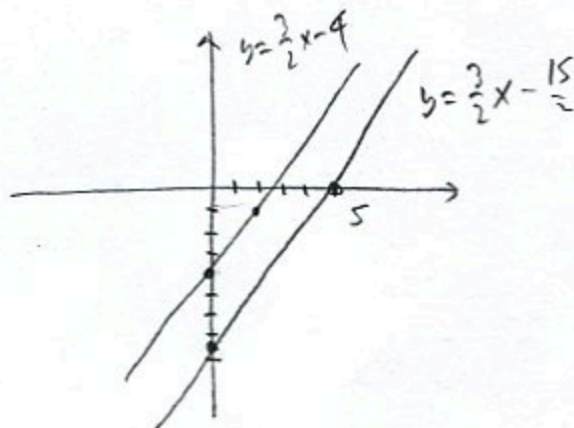$-2y = -3x + 8 \mid \div (-2)$

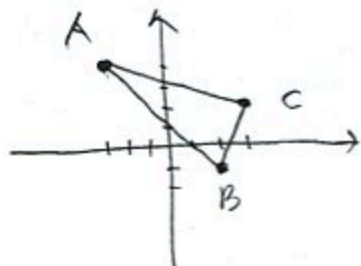$y = \dfrac{-3x}{-2} + \dfrac{8}{-2} = \dfrac{3}{2}x - 4$

$y - 0 =$

Slope of parallel line is same,

$m = \dfrac{3}{2}$. It passes through $(5,0)$

$y - 0 = \dfrac{3}{2}(x-5) = \quad y = \dfrac{3}{2}x - \dfrac{15}{2}$

$y = \dfrac{3}{2}x - 4$
$y = \dfrac{3}{2}x - \dfrac{15}{2}$

5. (7pts) Draw the triangle with vertices $A = (-3, 4)$, $B = (2, -1)$, $C = (3, 2)$. Use either slopes or lengths of sides (distance formula) to determine whether the triangle is a right triangle.

Find slopes of lines:

$m_{AC} = \dfrac{2-4}{3-(-3)} = \dfrac{-2}{6} = -\dfrac{1}{3}$ $\Bigg\}$ are opposite reciprocal, so lines AC and AB are perpendicular

$m_{BC} = \dfrac{2-(-1)}{3-2} = \dfrac{3}{1} = 3$

$m_{AB} = \dfrac{-1-4}{2-(-3)} = -\dfrac{5}{5} = -1$

Therefore, it is a right triangle

6. (9pts) Let $f(x) = \dfrac{\sqrt{2x-7}}{x^2 - 3x + 2}$. Find the following (simplify where appropriate).

$f(8) = \dfrac{\sqrt{2\cdot 8-7}}{8^2-3\cdot 8+2} = \dfrac{\sqrt{9}}{42} = \dfrac{3}{42} = \dfrac{1}{14}$

$f(2) = \dfrac{\sqrt{2\cdot 2-7}}{2^2-3\cdot 2+2} = \dfrac{\sqrt{-3}}{0}$ not defined

$f(-2x) = \dfrac{\sqrt{2(-2x)-7}}{(-2x)^2-3(-2x)+2}$

$= \dfrac{\sqrt{-4x-7}}{4x^2+6x+2}$

$f(u-3) = \dfrac{\sqrt{2(u-3)-7}}{(u-3)^2-3(u-3)+2}$

$= \dfrac{\sqrt{2u-6-7}}{u^2-6u+9-3u+9+2}$

$= \dfrac{\sqrt{2u-13}}{u^2-9u+20}$

7. (9pts) Find the domains of the functions below and write them using interval notation.

$f(x) = \dfrac{3x-1}{x^2+4x-5}$

Can't have $x^2+4x-5=0$

$(x+5)(x-1)=0$

$x=-5, 1$

$\xleftarrow{\hspace{1em}}\circ\!\!-\!\!-\!\!-\!\!-\!\!\circ\!\!-\!\!-\!\!-\!\!\rightarrow$
$\quad\quad -5 \quad\quad 1$

$(-\infty,-5)\cup(-5,1)\cup(1,\infty)$

$g(x) = \sqrt{16-6x}$

Must have: $16-6x \geq 0$

$16 \geq 6x$

$x \leq \dfrac{16}{6}$

$x \leq \dfrac{8}{3}$

$\xleftarrow{\hspace{2em}}\!]$
$\quad\quad \frac{8}{3}$
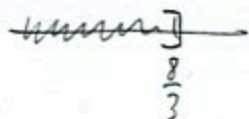
$\left(-\infty, \dfrac{8}{3}\right]$

8. (6pts) Solve and write the solution in interval notation.

$3x + 1 < 4$ or $2x - 5 > 9$

$3x < 3 \qquad 2x > 14$

$x < 1$ or $x > 7$ $\qquad (-\infty, 1) \cup (7, \infty)$

9. (10pts) The endpoints of a diameter of a circle are $(-1, 5)$ and $(3, -1)$.
a) Find the equation of the circle.
b) Draw the circle in the coordinate plane.

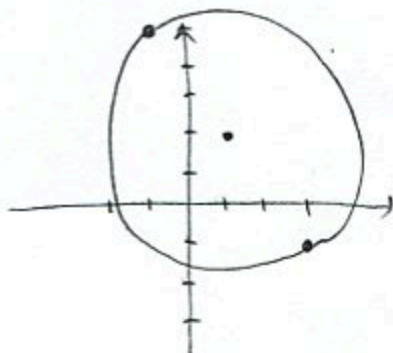center = midpoint of $(-1, 5)$ and $(3, -1)$

$= \left(\dfrac{-1+3}{2}, \dfrac{5-1}{2} \right) = \left(\dfrac{2}{2}, \dfrac{4}{2} \right) = (1, 2)$

radius = distance from $(1, 2)$ to $(3, -1)$

$= \sqrt{(3-1)^2 + (-1-2)^2}$

$= \sqrt{2^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$

Equation: $(x-1)^2 + (y-2)^2 = \sqrt{13}^2$

$(x-1)^2 + (y-2)^2 = 13$

10. (12pts) Zoe is considering which taxi company to use for a trip:
Ridewith charges a $4.00 for any ride up to one mile plus $1.75 per mile for miles past one.
Tripbuddy charges $2.25 per mile.
If Zoe rides more than one mile, for which number of miles traveled is Tripbuddy the better
option? Solve as an inequality.

x = numbers of miles traveled

cost of Ridewith: $4 + 1.75(x-1)$

Cost of Tripbuddy: $2.25x$

For trips between 1 and 4.5 miles
Tripbuddy is better.

Want to know when is

$2.25x \le 4 + 1.75(x-1)$

$2.25x \le 4 + 1.75x - 1.75$

$0.5x \le 2.25$

$x \le \dfrac{2.25}{0.5} =$

$x \le 4.5$

11. (14pts) Shepherd Billy looked away from his cow just when it started trotting away at 4 meters per second. Having realized what is happening 12 seconds later, he starts to chase the cow, running at 7 meters per second.

a) How long does Billy run until he catches up with the cow?

b) How far does he run until that moment?

Cow $\xrightarrow{\quad d, 4, t+12 \quad}$ $t =$ time Billy runs

Billy $\xrightarrow{\quad d, 7, t \quad}$ $t+12 =$ time cow trots

$d = 4(t+12)$

$d = 7t$

$4(t+12) = 7t$

$4t + 48 = 7t$

$48 = 3t$

$t = \frac{48}{3} = 16$

a) Billy catches up after 16 seconds

b) $d = 7 \cdot 16 = 112$ meters

Bonus (10pts) A university invests 1,400,000 at simple interest, part at 5%, half that amount at 3.5% and the rest at 5.5%. What is the most that the university can invest at 3.5% and still have at least $68,000 in interest per year? Solve as an inequality.

amount invested at 3.5% $= x$

_____ at 5% $= 2x$

_____ at 5.5% $= 1,400,000 - (2x + x) = 1,400,000 - 3x$

interest from 3.5% acct + interest from 5% acct + interest from 5.5% acct $\geq 68,000$

$x \cdot 0.035$ $+ 2x \cdot 0.05$ $+ (1,400,000 - 3x) \cdot 0.055 \geq 68,000$

$0.035x + 0.10x + 77000 - 0.165x \geq 68,000 \qquad | -77000$

$-0.03x \geq -9000$

$x \leq \frac{-9000}{-0.03}$

$x \leq 300,000$

At most 300,000 can be invested at 3.5%