**Calculus 1 — Exam 1**
**MAT 250, Spring 2017 — D. Ivanšić**

**Name:** ______________________

*Show all your work!*

**1.** (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim\limits_{x\to -3+} f(x) =$

$\lim\limits_{x\to 2^-} f(x) =$

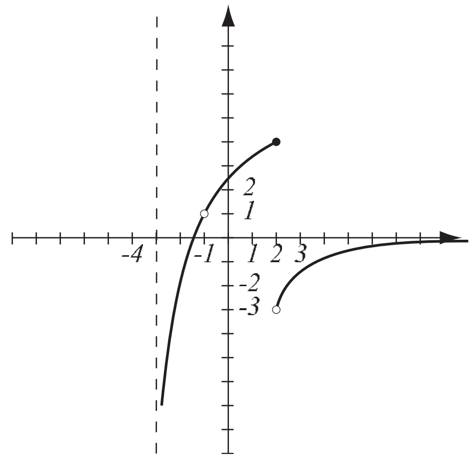
$\lim\limits_{x\to 2^+} f(x) =$

$\lim\limits_{x\to 2} f(x) =$

$\lim\limits_{x\to -1} f(x) =$

$\lim\limits_{x\to \infty} f(x) =$

List points where $f$ is not continuous and justify why it is not continuous at those points.




**2.** (6pts) Let $\lim\limits_{x\to 3} f(x) = 2$ and $\lim\limits_{x\to 3} g(x) = -1$. Use limit laws to find the limit below and show each step.

$$\lim_{x\to 3} \sqrt{x^3 f(x) - \frac{10}{g(x)}} =$$

**3.** (10pts) Find $\lim\limits_{x\to 0} x^2 \cdot \sqrt{7 + \sin\left(\frac{1}{x}\right)}$. Use the theorem that rhymes with honey-producing insects.

Find the following limits algebraically. Do not use the calculator.

**4.** (5pts) $\lim_{x \to 5} \dfrac{x^2 - 5x}{x^2 - 3x - 10} =$

**5.** (7pts) $\lim_{x \to 2} \dfrac{3 - \sqrt{x+7}}{x - 2} =$

**6.** (7pts) $\lim_{x \to 0} \dfrac{\tan(2x)}{x} =$

**7.** (7pts) $\lim_{x \to \infty} \dfrac{3x^2 - 5x + 4}{x^3 - 4x^2 + x - 7} =$

**8.** (6pts) $\lim_{x \to 2^+} \dfrac{x - 6}{4 - 2x} =$

**9.** (14pts) The equation $x^2 + 4x = 2^x + 5$ is given.
a) Use the Intermediate Value Theorem to show it has a solution in the interval $(0, 3)$.
b) Use your calculator to find an interval of length at most 0.01 that contains a solution of the equation. Then use the Intermediate Value Theorem to justify why your interval contains the solution.

**10.** (10pts) Consider the limit $\lim_{x \to 1} \frac{\log x}{x - 1}$. Use your calculator (careful with entering the denominator!) to estimate this limit with accuracy 4 decimal points. Write a table of values that will justify your answer.

| $x$ | $\frac{\log x}{x-1}$ |
|---|---|
| | |

| $x$ | $\frac{\log x}{x-1}$ |
|---|---|
| | |

**11.** (12pts) Consider the function defined below. Find a value for $c$ that makes the function continuous.

$$f(x) = \begin{cases} x^2 + \dfrac{cx}{16}, & \text{if } x \leq 4 \\ \dfrac{cx - 4c}{x^2 - 16}, & \text{if } x > 4. \end{cases}$$

**Bonus.** (10pts) Find the limit algebraically.

$$\lim_{x \to \infty} (\sqrt{x^2 + 5x + 2} - x)$$

**Calculus 1 — Exam 2**
**MAT 250, Spring 2017 — D. Ivanšić**

**Name:** ____________________

*Show all your work!*

Differentiate and simplify where appropriate:

**1.** (6pts) $\dfrac{d}{dx}\left(2x^4+\dfrac{8}{x^5}-\dfrac{1}{\sqrt[9]{x^4}}-\pi^5\right)=$

**2.** (5pts) $\dfrac{d}{du}\sin u\tan u=$

**3.** (6pts) $\dfrac{d}{dx}\dfrac{3x-1}{x^3+2x^2+1}=$

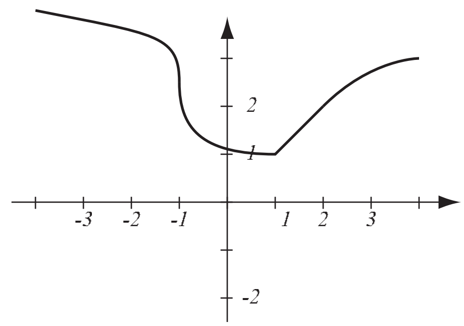
**4.** (6pts) $\dfrac{d}{d\theta}\left(\cos^2\theta-\cos(2\theta)\right)=$

**5.** (6pts) $\dfrac{d}{dx}\sin\sqrt{x^2+3x+5}=$

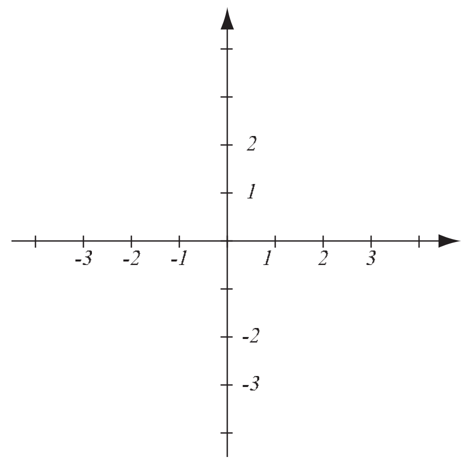
**6.** (7pts) The limit at right represents a derivative $f'(a)$.
a) State $f$ and $a$.
b) Evaluate $f'(a)$ using differentiation rules — this gives you the limit.

$$\lim_{x\to 2}\frac{x^5-32}{x-2}$$

**7.** (10pts) The graph of the function $f(x)$ is shown at right.




a) Where is $f(x)$ not differentiable? Why?
b) Use the graph of $f(x)$ to draw an accurate graph of $f'(x)$.




**8.** (14pts) Let $f(x) = \dfrac{4}{x}$.
a) Use the limit definition of the derivative to find the derivative of the function.
b) Check your answer by taking the derivative of $f$ using differentiation rules.
c) Write the equation of the tangent line to the curve $y = f(x)$ at point $(1, 4)$.

**9.** (8pts) Let $g(x) = \dfrac{x}{f(x)}$ and $h(x) = f(f(x))$.
a) Find the general expressions for $g'(x)$ and $h'(x)$.
b) Use the table of values at right to find $g'(2)$ and $h'(4)$.

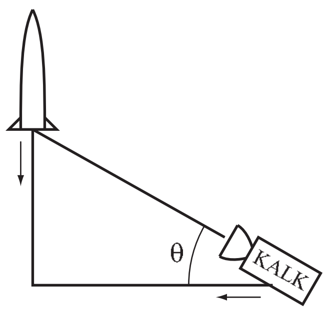
| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $f(x)$ | 2 | 3 | 7 | 1 |
| $f'(x)$ | -2 | 1 | -3 | 3 |

**10.** (6pts) An egg thrown upwards from height 3 meters has position given by the formula $s(t) = -5t^2 + 8t + 3$.
a) Write the formula for the velocity of the egg at time $t$.
b) When does the egg reach its maximum height?
c) What is the initial velocity of the egg?

**11.** (10pts) Use implicit differentiation to find $y'$.

$\sin(x + y) = x^2 + xy + y^2$

**12.** (16pts) Late to the event, the crew of TV-station KALK is rushing on a straight road to a vertical landing of a new rocket. Their rooftop camera is aimed at the rocket. KALK's car is approaching the landing site at 35 meters per second and the rocket, located directly above the landing site, is descending at 15 meters per second. At what rate is the angle of elevation $\theta$ of the line of sight to the rocket changing when the rocket is 3000 meters above ground, and the car is 4000 meters from the landing site? Is the camera tilting lower or higher at that moment? *(Hint: sohcahtoa.)*




**Bonus.** (10pts) It is common knowledge that the tangent line to a circle is perpendicular to the radius at the point of tangency. Show this fact using slopes of the tangent line and the radius line through point $(a, b)$ of the circle $x^2 + y^2 = r^2$. *(Hint: implicit differentiation will make it easier here.)*

**Calculus 1 — Exam 3**
**MAT 250, Spring 2017 — D. Ivanšić**

**Name:** ____________________

*Show all your work!*

Differentiate and simplify where appropriate:

**1.** (5pts) $\dfrac{d}{dx}\,(x^2+4x-5)e^x =$

**2.** (4pts) $\dfrac{d}{dx}\,\ln(\sqrt{\cos x+1}) =$

**3.** (7pts) $\dfrac{d}{dt}\,\dfrac{2^t+t^2}{t} =$

**4.** (7pts) $\dfrac{d}{dx}\,\ln\dfrac{1+\sin x}{1-\sin x} =$

**5.** (7pts) $\dfrac{d}{dw}\,\dfrac{\arccos w}{\sqrt{1-w^2}} =$

**6.** (10pts) Use logarithmic differentiation to find the derivative of $y=(\cos x)^{\sin x}$.

Find the limits algebraically. Graphs of basic functions will help, as will L'Hospital's rule, where appropriate.

**7.** (2pts) $\lim_{x \to -\infty} 1.2^x =$

**8.** (6pts) $\lim_{x \to \infty} \arctan\left(\frac{x^2+4}{x+1}\right) =$

**9.** (7pts) $\lim_{x \to 0} \frac{\ln(x+1) - x}{x^2} =$

**10.** (8pts) $\lim_{x \to \infty} x^2 \sin\frac{1}{x} =$

**11.** (9pts) $\lim_{x \to \infty} (e^x + x)^{\frac{1}{x}} =$

**12.** (10pts) Let $f(x) = e^x$.
a) Write the linearization of $f(x)$ at $a = 0$.
b) Use the linearization to estimate $e^{0.15}$ and compare to the calculator value of 1.161834.

**13.** (10pts) A square field is measured to have side length of 16 kilometers, with maximum error 40 meters. Use differentials to estimate the maximum possible error, the relative error and the percentage error when computing the area of the field.

**14.** (8pts) Let $f(x) = x^3 + 4x^2 - 7x + 6$. Use the theorem on derivatives of inverses to find $(f^{-1})'(4)$.

**Bonus.** (10pts) We have stated that $0^0$ is an indeterminate form, yet most limits of this type come out equal to 1. The purpose of this problem is to find examples where $0^0 \neq 1$.

a) Show that $\lim_{x \to 0} e^{-\frac{1}{x^2}} = 0$.

b) Find three different functions $g(x)$ so that $\lim_{x \to 0} g(x) = 0$ in each case, but $\lim_{x \to 0} (e^{-\frac{1}{x^2}})^{g(x)}$ equals $\frac{1}{e}$, 0, and $\frac{1}{2}$, respectively. Thus, by setting $f(x) = e^{-\frac{1}{x^2}}$, you will get three limits $\lim_{x \to 0} f(x)^{g(x)}$ of type $0^0$ that do not equal 1.

**Calculus 1 — Exam 4**
**MAT 250, Spring 2017 — D. Ivanšić**

**Name:** ______________________________

*Show all your work!*

**1.** (30pts) Let $f(x) = (x^2 + x + 2)e^x$. Draw an accurate graph of $f$ by following the guidelines.
a) Find the intervals of increase and decrease, and local extremes.
b) Find the intervals of concavity and points of inflection.
c) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
d) Use information from a)–c) to sketch the graph.

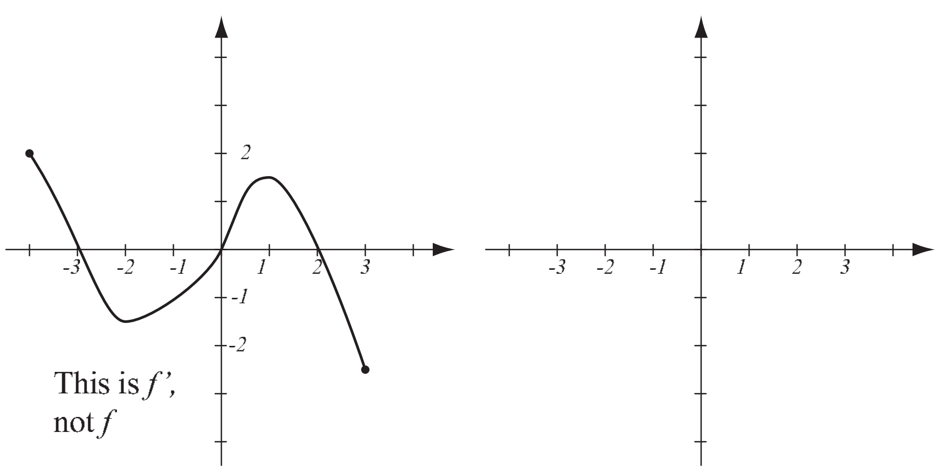
**2.** (14pts) Let $f(x) = \sin^2 x - \cos x$. Find the absolute minimum and maximum values of $f$ on the interval $[0, \pi]$.

**3.** (18pts) Let $f$ be continuous on $[-4, 3]$. The graph of its derivative $f'$ is drawn below. Use the graph to answer (sign charts may help):

a) What are the intervals of increase and decrease of $f$? Where does $f$ have a local minimum or maximum?

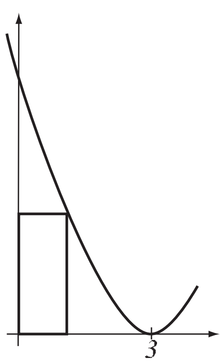
b) What are the intervals of concavity of $f$? Where does $f$ have inflection points?

c) Use the information gathered in a) and b) to sketch the graph of $f$ at right, if $f(-4) = 0$.



**4.** (16pts) Consider $f(x) = x^2 - 3x + 5$ on the interval $[1, 4]$.
a) Verify that the function satisfies the assumptions of the Mean Value Theorem.
b) Find all numbers $c$ that satisfy the conclusion of the Mean Value Theorem.

**5.** (22pts) Consider a rectangle with sides on the $x$- and $y$-axes whose one vertex lies on the parabola $y = (x - 3)^2$ and is enclosed in the region between the axes and the parabola. Among all such rectangles, find the one with the biggest area.



**Bonus.** (10pts) Draw a function, if possible, that satisfies the given conditions. Justify if such a function is not possible.
a) $f$ defined on $[1, 4]$, has a local maximum but no absolute maximum.
b) $f$ continuous on $[1, 4]$, has a local minimum but no absolute maximum.
c) $f$ defined on $[1, 4)$, has no local minimum nor maximum, and has no absolute minimum nor maximum.

**Calculus 1 — Exam 5**
**MAT 250, Spring 2017 — D. Ivanšić**

**Name:** ______________________________

*Show all your work!*

Find the following antiderivatives.

**1.** (3pts) $\displaystyle\int \frac{1}{\sqrt[4]{x^3}}\,dx =$

**2.** (3pts) $\displaystyle\int \frac{4}{1+x^2}\,dx =$

**3.** (3pts) $\displaystyle\int \cos\left(2x+\frac{\pi}{2}\right)\,dx =$

**4.** (7pts) $\displaystyle\int (t^3-4t^2)\sqrt{t}\,dt =$

**5.** (7pts) Find $f(x)$ if $f'(x) = 2e^{4x} + \sec x \tan x$ and $f(0) = 3$.

**6.** (6pts) Write using sigma notation:

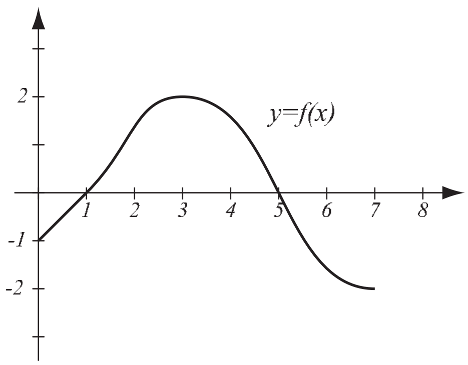
$-3+6-9+12-15+\cdots+300-303 =$

**7.** (15pts) The function $f(x) = 2^x$ is given on the interval $[-1, 1]$.
a) Write the Riemann sum $M_6$ for this function with six subintervals, taking sample points to be midpoints. Do not evaluate the expression.
b) Illustrate with a diagram, where appropriate rectangles are clearly visible. What does $M_6$ represent?

**8.** (13pts) Find $\int_0^2 2x - 1\,dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture.
b) Using the Evaluation Theorem.

**9.** (7pts) The graph of a function $f$ is shown. Put the four numbers 0, $a$, $b$, $c$ in increasing order and justify your reasoning.

$a = \int_0^1 f(x)\,dx$ $\qquad$ $b = \int_0^5 f(x)\,dx$ $\qquad$ $c = \int_0^7 f(x)\,dx$




Use the substitution rule in the following integrals:

**10.** (8pts) $\displaystyle\int (x+2)\sqrt[9]{x^2+4x+3}\,dx =$

**11.** (10pts) $\displaystyle\int_1^{e^\pi} \frac{\sin(\ln x)}{x}\,dx =$

**12.** (8pts) $\displaystyle\int_2^4 \frac{2x-6}{\cot(x^2-6x+5)}\,dx =$

**13.** (10pts) The rate at which a river's water level is changing is $t^2 - 4t - 21$ feet per day.
a) Use the Net Change Theorem to find by how much the river water level has changed from $t = 6$ to $t = 9$.
b) If at time $t = 6$ the river water level was 32 feet, what is it at time $t = 9$?

**Bonus.** (10pts) Evaluate. A picture will help.

$$\int_{\frac{\pi}{6}}^{\frac{4\pi}{3}} |\sin x|\, dx =$$

| Calculus 1 — Final Exam<br>MAT 250, Spring 2017 — D. Ivanšić |
|---|

**Name:** ____________________________

*Show all your work!*

**1.** (15pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

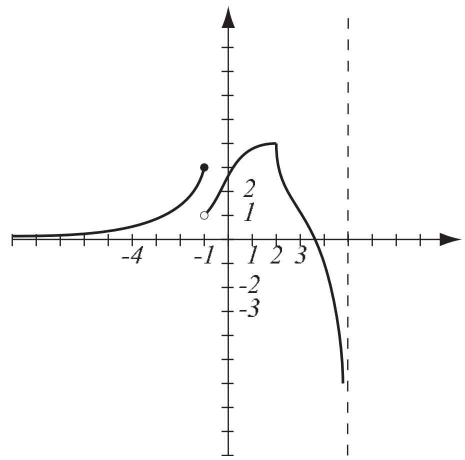
$\lim\limits_{x\to -1^-} f(x) =$

$\lim\limits_{x\to -1^+} f(x) =$

$\lim\limits_{x\to -1} f(x) =$

$\lim\limits_{x\to 5^-} f(x) =$

$\lim\limits_{x\to -\infty} f(x) =$




List points where $f$ is not continuous and explain why.

List points where $f$ is not differentiable and explain why.

Find the following limits algebraically. Do not use L'Hospital's rule.

**2.** (6pts) $\lim\limits_{x\to\infty} \dfrac{x^2-3x+4}{3x^2-4x-2} =$

**3.** (6pts) $\lim\limits_{x\to 3^-} \dfrac{x^2+5x}{2x-6} =$

**4.** (6pts) The equation $x^2 + \cos x = 5 - 2x$ is given. Use the Intermediate Value Theorem to show it has a solution.

**5.** (14pts) The curve with equation $\sin x \sin y = x^2 - y^2$ is given.
a) Use implicit differentiation to find $y'$.
b) Find the equation of the tangent line at $(\pi, \pi)$.

**6.** (8pts) A square field is measured to have side length of 5 kilometers, with maximum error 25 meters. Use differentials to estimate the maximum possible error, the relative error and the percentage error when computing the area of the field.

**7.** (26pts) Let $f(x) = (x^2 + 1)e^x$. Draw an accurate graph of $f$ by following the guidelines.
a) Find the intervals of increase and decrease, and local extremes.
b) Find the intervals of concavity and points of inflection.
c) Find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$. Use L'Hospital's rule where necessary.
d) Use information from a)–c) to sketch the graph.

**8.** (6pts) Find $f(x)$ if $f'(x) = \sqrt[3]{x^4} + \dfrac{1}{x}$ and $f(1) = 5$.

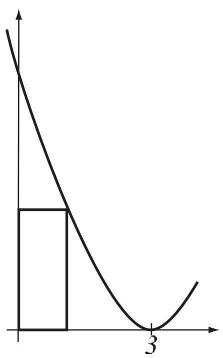
**9.** (13pts) Find $\int_{-3}^{1} 2x + 1\, dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture.
b) Using the Evaluation Theorem.

Use the substitution rule in the following integrals:

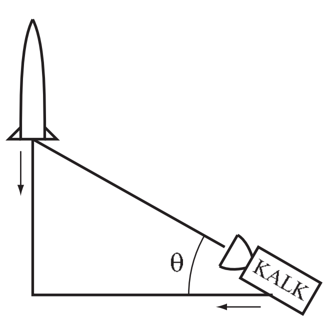
**10.** (7pts) $\displaystyle\int (e^x + 1)\sec^2(e^x + x)\, dx =$

**11.** (9pts) $\displaystyle\int_1^3 \frac{6x+3}{\sqrt{x^2+x+4}}\, dx =$

**12.** (18pts) Consider a rectangle with sides on the $x$- and $y$-axes whose one vertex lies on the parabola $y = (x-3)^2$ and is enclosed in the region between the axes and the parabola. Among all such rectangles, find the one with the biggest area.




**13.** (16pts) Late to the event, the crew of TV-station KALK is rushing on a straight road to a vertical landing of a new rocket. Their rooftop camera is aimed at the rocket. KALK's car is approaching the landing site at 20 meters per second and the rocket, located directly above the landing site, is descending at 12 meters per second. At what rate is the angle of elevation $\theta$ of the line of sight to the rocket changing when the rocket is 200 meters above ground, and the car is 1000 meters from the landing site? Is the camera tilting lower or higher at that moment? *(Hint: sohcahtoa.)*



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**Bonus.** (7pts) Find the limit algebraically.

$$\lim_{x \to \infty} (\sqrt{x^2 + 5x + 2} - x)$$