

Calculus 1 — Final Exam
MAT 250, Spring 2015 — D. Ivanšić

Name: _____
Show all your work!

1. (13pts) Use the graph of the function with domain $(-5, 7]$ to answer the following. Justify your answer if a limit does not exist.

$$\lim_{x \rightarrow -5^+} f(x) =$$

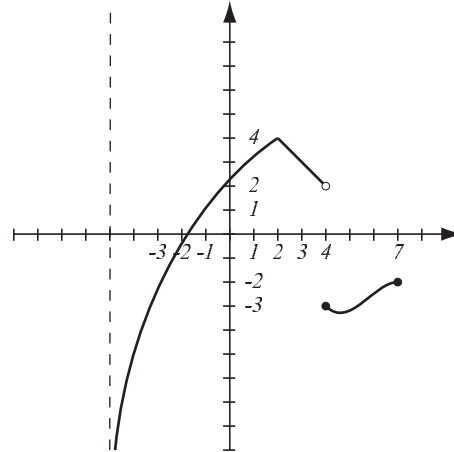
$$\lim_{x \rightarrow 4^+} f(x) =$$

$$\lim_{x \rightarrow 4^-} f(x) =$$

$$\lim_{x \rightarrow 4} f(x) =$$

$$\lim_{x \rightarrow -1} f(x) =$$

List points where f is not differentiable and explain why



Find the following limits algebraically. Do not use L'Hospital's rule

2. (5pts) $\lim_{x \rightarrow \infty} \frac{x^2 - 3x + 2}{7x^3 - x^2 + 5} =$

3. (4pts) $\lim_{x \rightarrow 3^+} \frac{2x - 1}{3 - x} =$

Use L'Hospital's rule to find the limit:

4. (4pts) $\lim_{x \rightarrow 0} \frac{e^{2x} - e^x}{3x} =$

5. (8pts) Use the Intermediate Value Theorem to show that the equation $x^2 + 1 = x^3 - 5x$ has at least one solution.

6. (16pts) Consider the curve with the equation $x + y + x^2y^2 = \cos y$.

a) Use implicit differentiation to find y' .

b) Find the equation of the tangent line to the curve at point $(1, 0)$.

7. (10pts) In a rectangle, length is 5 meters longer than the width. If width is measured to be 7 meters with maximum error 2 centimeters, use differentials to estimate the maximum possible error and the relative error when computing the area of the rectangle.

8. (24pts) Let $f(x) = \ln(x^2 + 1)$. Draw an accurate graph of f by following the guidelines.
- Find the intervals of increase and decrease, and local extremes.
 - Find the intervals of concavity and points of inflection.
 - Find $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$.
 - Use information from a)–d) to sketch the graph.

9. (12pts) Let $f(x) = \sin^2 x + \sin x$. Find the absolute minimum and maximum values of f on the interval $[0, 2\pi]$.

10. (6pts) Find $f(x)$ if $f'(x) = e^{3x+6} + \frac{8}{x^5}$ and $f(-2) = -3$.

Use the substitution rule to find the integral:

11. (10pts) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{2 + \cos x} dx =$

12. (10pts) The rate at which people are arriving to Lovett auditorium is $40 - 10\sqrt{t}$ people per minute.

a) Use the Net Change Theorem to find how many people arrived from $t = 0$ to $t = 9$.

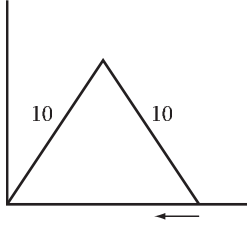
b) If at time $t = 0$ there were 150 people in the auditorium, how many were there at time $t = 9$?

13. (12pts) Consider the integral $\int_0^4 x^2 + 2x - 15 \, dx$.

a) Use a picture and the “area” interpretation of the integral to determine whether this integral is positive or negative.

b) Use the Evaluation Theorem to find the integral and verify your conclusion from a).

14. (16pts) A folding ladder whose sides are 10ft long has one end against a wall. If the other end is pushed toward the wall at rate $1/4$ foot per second, how fast is the top of the ladder rising when the pushed end is 6 feet away from the wall?



Bonus. (15pts) Find the limit. (Note: for small $x > 0$, $\ln x < 0$, so we need a minus to ensure that the base is a positive number).

$$\lim_{x \rightarrow 0^+} (-\ln x)^{\ln(x+1)} =$$