

Calculus 1 — Exam 4
MAT 250, Spring 2015 — D. Ivanšić

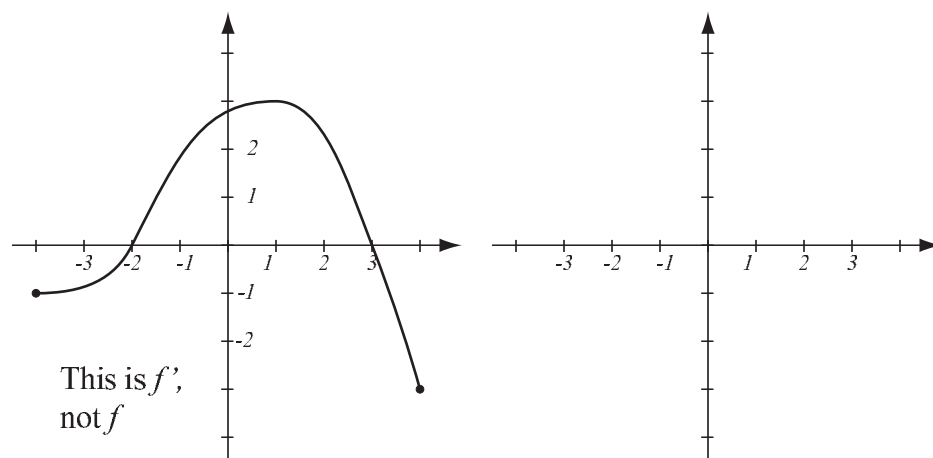
Name: _____
Show all your work!

1. (30pts) Let $f(x) = \frac{x^2}{x^2 + 1}$. Draw an accurate graph of f by following the guidelines.
- Find the intervals of increase and decrease, and local extremes.
 - Find the intervals of concavity and points of inflection.
 - Find $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$.
 - Use information from a)–d) to sketch the graph.

2. (14pts) Let $f(x) = 24x^{\frac{1}{2}} - 2x^{\frac{3}{2}}$. Find the absolute minimum and maximum values of f on the interval $[1, 9]$.

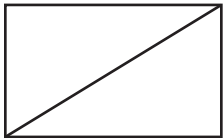
3. (16pts) Let f be continuous on $[-4, 4]$. The graph of its derivative f' is drawn below. Use the graph to answer (sign charts may help):

- What are the intervals of increase and decrease of f ? Where does f have a local minimum or maximum?
- What are the intervals of concavity of f ? Where does f have inflection points?
- Use the information gathered in a) and b) to sketch the graph of f at right, if $f(-4) = 0$.



4. (16pts) Let $f(x) = \sin^2 x$, $0 \leq x \leq 2\pi$. Find the intervals of concavity and points of inflection for f .

5. (24pts) Among all rectangles of area 100 square meters, find the one which has the shortest diagonal.



- Bonus.** (10pts) Suppose $f(x) > 0$ and f is concave up. Let $g(x) = (f(x))^2$.
- Find the expression for $g''(x)$.
 - Show that g is concave up.