

Calculus 1 — Exam 3
MAT 250, Spring 2015 — D. Ivanšić

Name: _____
Show all your work!

Differentiate and simplify where appropriate:

1. (3pts) $\frac{d}{dx} e^{x^2+3x-1} =$

2. (4pts) $\frac{d}{dx} \ln(\tan^2 x) =$

3. (6pts) $\frac{d}{dt} \frac{t^2 - 3t}{7^t} =$

4. (7pts) $\frac{d}{dx} \ln \frac{\sin x + \cos x}{\sin x - \cos x} =$

5. (8pts) $\frac{d}{du} \left(u \arctan u - \frac{1}{2} \ln(1 + u^2) \right) =$

6. (10pts) Use logarithmic differentiation to find the derivative of $y = x^{\sqrt{x}}$.

7. (4pts) Draw the graphs of e^x , $\ln x$ and $\arctan x$ (each in its coordinate system).

Find the limits algebraically. Graphs of basic functions will help, as will L'Hospital's rule, where appropriate.

8. (2pts) $\lim_{x \rightarrow -\infty} 5^x =$

9. (6pts) $\lim_{x \rightarrow 0^+} \arctan \left(4 - \frac{1}{x} \right) =$

10. (6pts) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} =$

11. (6pts) $\lim_{x \rightarrow 0^+} x^3 \ln x =$

12. (10pts) $\lim_{x \rightarrow \infty} (x^2 + 3x - 1)^{\frac{1}{x}} =$

13. (10pts) Let $f(x) = \sqrt[3]{x}$.

a) Write the linearization of $f(x)$ at $a = 8$.

b) Use the linearization to estimate $\sqrt[3]{8.3}$ and compare to the calculator value of 2.024694.

14. (10pts) Radius of a sphere r is measured to be 10 meters, with maximum error 5 centimeters. Use differentials to estimate the maximum possible error, the relative error and the percentage error when computing the surface area A of the sphere ($A = 4\pi r^2$, leave your answer in terms of π).

15. (8pts) Let $f(x) = e^x + 3x + 4$. Use the theorem on derivatives of inverses to find $(f^{-1})'(5)$.

Bonus. (10pts) Find the limit. (Note: for small $x > 0$, $\ln x < 0$, so we need a minus to ensure that the base is a positive number).

$$\lim_{x \rightarrow 0^+} (-\ln x)^{\ln(x+1)} =$$