

Calculus 1 — Exam 1
 MAT 250, Spring 2015 — D. Ivanšić

Name: _____
Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$$\lim_{x \rightarrow -3} f(x) =$$

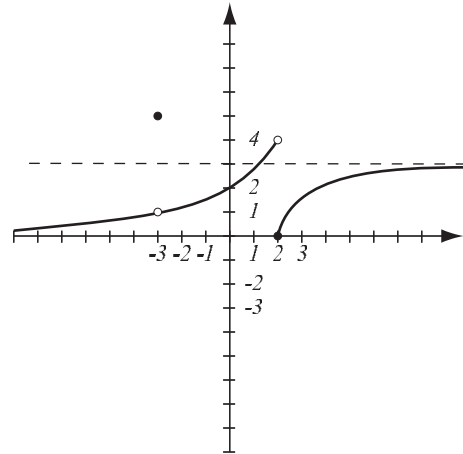
$$\lim_{x \rightarrow 2^+} f(x) =$$

$$\lim_{x \rightarrow 2^-} f(x) =$$

$$\lim_{x \rightarrow 2} f(x) =$$

$$\lim_{x \rightarrow \infty} f(x) =$$

$$\lim_{x \rightarrow -\infty} f(x) =$$



List points where f is not continuous and justify why it is not continuous at those points.

2. (4pts) Briefly explain why the function $f(x) = \frac{x+3}{3x-2}$ is continuous on its domain.

3. (10pts) Find $\lim_{x \rightarrow 0} x^4 \left(7 + \cos\left(\frac{1}{x^3}\right) \right)$. Use the theorem that rhymes with what a doctor may cure.

Find the following limits algebraically. Do not use the calculator.

4. (5pts) $\lim_{x \rightarrow 7} \frac{x - 7}{x^2 - 2x - 35} =$

5. (7pts) $\lim_{x \rightarrow 16} \frac{\sqrt{x} - 4}{x - 16} =$

6. (7pts) $\lim_{x \rightarrow 0} \frac{\sin(3x)}{5x} =$

7. (7pts) $\lim_{x \rightarrow \infty} \frac{4x^3 - 3x^2 - 7x + 2}{7x^3 - x^2 + 5x} =$

8. (6pts) $\lim_{x \rightarrow 5^+} \frac{4 - 2x}{x - 5} =$

9. (10pts) Use the Intermediate Value Theorem to show that the equation $x^3 + 2x = 4\sqrt{x} + 2$ has at least one solution.

10. (10pts) Consider the limit $\lim_{x \rightarrow 0} \frac{5^x - 1}{x}$. Use your calculator to estimate this limit with accuracy 4 decimal points. Write a table of values that will justify your answer.

x	$\frac{5^x - 1}{x}$	x	$\frac{5^x - 1}{x}$

11. (4pts) Consider the limit below, representing a derivative $f'(a)$: find f and a .

$$\lim_{h \rightarrow 0} \frac{(3 + h)^4 - 81}{h}$$

12. (14pts) The amount of water (in gallons) in a 100-gallon tank that is draining at the bottom is given by $V(t) = t^2 - 20t + 100$, where t is in minutes, $0 \leq t \leq 10$.

- a) What is the average rate of draining from $t = 2$ to $t = 5$? What are the units?
- b) What is the instantaneous rate of draining when $t = 2$? What are the units?

Bonus. (10pts) Consider the limit $\lim_{x \rightarrow \sqrt{3}} \frac{x^2 - 3}{x - \sqrt{3}}$.

- a) Use your calculator to estimate this limit with accuracy 4 decimal points. Write a table of values that will justify your answer.
- b) Find the limit algebraically.

x	$\frac{x^2 - 3}{x - \sqrt{3}}$