Calculus 1 — Exam 5
MAT 250, Spring 2017 — D. Ivanšić

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Show all your work!

Find the following antiderivatives.

1. (3pts) $\int \frac{1}{\sqrt[4]{x^3}}\,dx =$ $\int x^{-\frac{3}{4}}\,dx = 4x^{\frac{1}{4}} + C = 4\sqrt[4]{x} + C$

2. (3pts) $\int \frac{4}{1+x^2}\,dx =$ $4\arctan x + C$

3. (3pts) $\int \cos\left(2x + \frac{\pi}{2}\right)\,dx =$ $\frac{\sin\left(2x+\frac{\pi}{2}\right)}{2} + C$

4. (7pts) $\int (t^3 - 4t^2)\sqrt{t}\,dt =$ $\int (t^3-4t^2)\cdot t^{1/2}\,dt = \int t^{\frac{7}{2}} + 4t^{\frac{5}{2}}\,dt$

$= \frac{2}{9}t^{\frac{9}{2}} + 4\cdot\frac{2}{7}t^{\frac{7}{2}} = \frac{2}{9}t^{\frac{9}{2}} + \frac{8}{7}t^{\frac{7}{2}} + C$

5. (7pts) Find $f(x)$ if $f'(x) = 2e^{4x} + \sec x \tan x$ and $f(0) = 3$.

$f'(x) = 2e^{4x} + \sec x \tan x$

$f(x) = 2\cdot\frac{e^{4x}}{4} + \sec x + C$

$= \frac{e^{4x}}{2} + \sec x + C$

$= \frac{1}{\cos 0}$

$3 = f(0) = \frac{e^0}{2} + \sec 0 + C$

$3 = \frac{1}{2} + 1 + C \quad \Big| -\frac{3}{2}$

$C = \frac{3}{2}, \quad f(x) = \frac{e^{4x}}{2} + \sec x + \frac{3}{2}$

6. (6pts) Write using sigma notation:

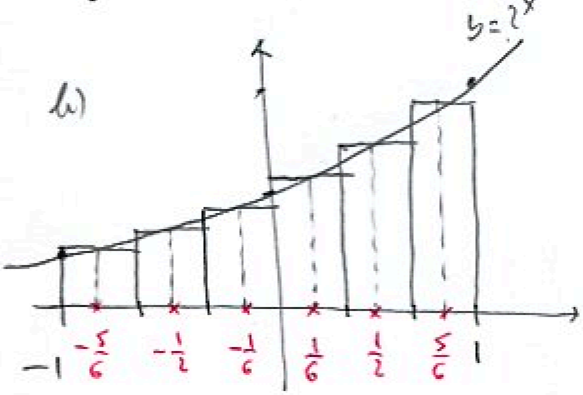
$-3 + 6 - 9 + 12 - 15 + \cdots + 300 - 303 =$ $\sum_{i=1}^{101} (-1)^i 3i$

multiples of 3, $3i$

7. (15pts) The function $f(x) = 2^x$ is given on the interval $[-1, 1]$.
a) Write the Riemann sum M_6 for this function with six subintervals, taking sample points to be midpoints. Do not evaluate the expression.
b) Illustrate with a diagram, where appropriate rectangles are clearly visible. What does M_6 represent?

$\Delta x = \frac{1}{3}$

b)

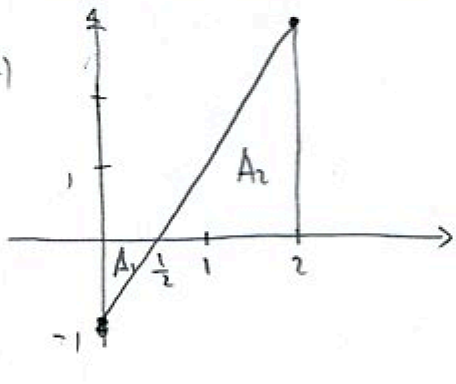


a) $M_6 = \frac{1}{3}\left(2^{-\frac{5}{6}} + 2^{-\frac{1}{2}} + 2^{-\frac{1}{6}} + 2^{\frac{1}{6}} + 2^{\frac{1}{2}} + 2^{\frac{5}{6}}\right)$

M_6 = sum of areas of rectangles

8. (13pts) Find $\int_0^2 2x - 1\, dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture.
b) Using the Evaluation Theorem.

a)

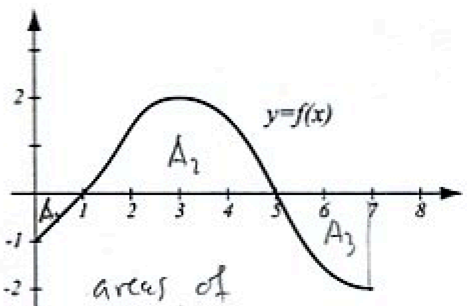


$$\int_0^2 2x-1\, dx = -A_1 + A_2$$
$$= -\frac{1}{2}\cdot\frac{1}{2}\cdot 1 + \frac{1}{2}\cdot\frac{3}{2}\cdot 3$$
$$= -\frac{1}{4} + \frac{9}{4} = \frac{8}{4} = 2$$

b) $\int_0^2 2x-1\, dx = (x^2 - x)\Big|_0^2 = (2^2-2)-(0-0) = 2$

9. (7pts) The graph of a function f is shown. Put the four numbers 0, a, b, c in increasing order and justify your reasoning.

$a = \int_0^1 f(x)\,dx$ $\qquad$ $b = \int_0^5 f(x)\,dx$ $\qquad$ $c = \int_0^7 f(x)\,dx$



areas of parts $A_1, A_2, A_3 > 0$

$a = -A_1 < 0$

$b = -A_1 + A_2 > 0$, since $A_1 < A_2$

$c = -A_1 + A_2 - A_3 = b - A_3 < b$

Also, $c = A_2 - (A_1 + A_3) > 0$ since $A_1 + A_3 < A_2$

Thus: $a < 0 < c < b$

Use the substitution rule in the following integrals:

10. (8pts) $\int (x+2)\sqrt[9]{x^2+4x+3}\,dx =$ $\left[\begin{array}{l} u = x^2+4x+3 \\ du = 2x+4\,dx \\ \frac{du}{2} = x+2\,dx \end{array}\right] = \int \sqrt[9]{u} \cdot \frac{du}{2}$ $\quad u^{1/9}$

$= \frac{1}{2} \cdot \frac{9}{10} u^{\frac{10}{9}} = \frac{9}{20}(x^2+4x+3)^{10/9} + C$

11. (10pts) $\int_1^{e^\pi} \frac{\sin(\ln x)}{x}\,dx =$ $\left[\begin{array}{ll} u = \ln x & x = e^\pi,\ u = \pi \\ du = \frac{1}{x}\,dx & x = 1,\ u = 0 \end{array}\right]$

$= \int_0^\pi \sin u\,du = -\cos u \Big|_0^\pi = -(\cos\pi - \cos 0) = -(-1-1) = 2$

12. (8pts) $\int_2^4 \frac{2x-6}{\cot(x^2-6x+5)}\,dx =$ $\left[\begin{array}{lll} u = x^2-6x+5 & x = 4, & u = 16-24+5 = -3 \\ du = 2x-6\,dx & x = 2, & u = 4-12+5 = -3 \end{array}\right]$

$= \int_{-3}^{-3} \frac{du}{\cot u} = 0$ since the interval of integration has no width.

13. (10pts) The rate at which a river's water level is changing is $t^2 - 4t - 21$ feet per day.
a) Use the Net Change Theorem to find by how much the river water level has changed from $t = 6$ to $t = 9$.
b) If at time $t = 6$ the river water level was 32 feet, what is it at time $t = 9$?

$$\ell'(t) = t^2 - 4t - 21$$

$$\ell(9) - \ell(6) = \int_6^9 t^2 - 4t - 21 \, dt = \left(\frac{t^3}{3} - 2t^2 - 21t\right)\Big|_6^9 = \frac{1}{3}(9^3 - 6^3) - 2(9^2 - 6^2) - 21(9-6)$$

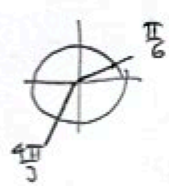
$$= \frac{1}{3}(729 - 216) - 2(81 - 36) - 21 \cdot 3$$

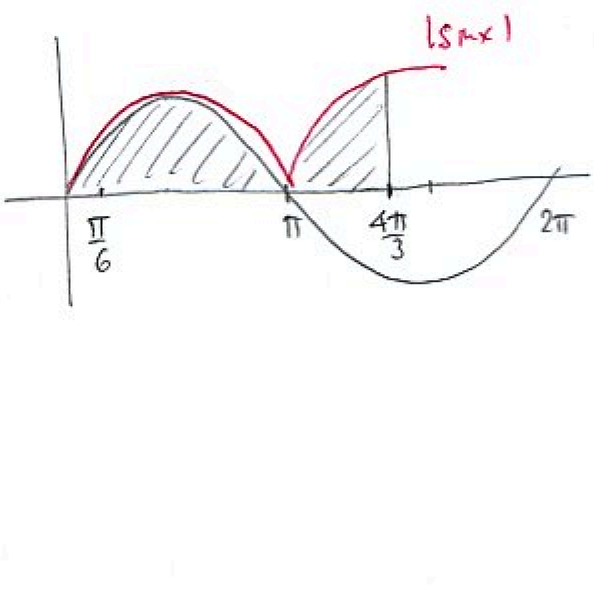
$$= \frac{1}{3} \cdot 513 - 2 \cdot 45 - 63 = 171 - 153 = 18 \text{ feet}$$

b) $\ell(9) = \ell(6) + (\ell(9) - \ell(6)) = 32 + 18 = 50$ feet

Bonus. (10pts) Evaluate. A picture will help.

$$\int_{\frac{\pi}{6}}^{\frac{4\pi}{3}} |\sin x| \, dx = \int_{\pi/6}^{\pi} \sin x \, dx + \int_{\pi}^{\frac{4\pi}{3}} -\sin x \, dx = (-\cos x)\Big|_{\pi/6}^{\pi} + \cos x\Big|_{\pi}^{\frac{4\pi}{3}}$$





$$= -\left(\cos\pi - \cos\frac{\pi}{6}\right) + \cos\frac{4\pi}{3} - \cos\pi$$

$$= -\left(-1 - \frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right) - (-1)$$

$$= 1 + \frac{\sqrt{3}}{2} - \frac{1}{2} + 1 = \frac{3}{2} + \frac{\sqrt{3}}{2}$$