Calculus 1 — Exam 4	**Name:** Saul Ocean
MAT 250, Spring 2017 — D. Ivanšić	*Show all your work!*

1. (30pts) Let $f(x) = (x^2 + x + 2)e^x$. Draw an accurate graph of f by following the guidelines.

a) Find the intervals of increase and decrease, and local extremes.

b) Find the intervals of concavity and points of inflection.

c) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

d) Use information from a)–c) to sketch the graph.

$f'(x) = (2x+1)e^x + (x^2+x+2)e^x$

$= (x^2+3x+3)e^x$

$f''(x) = (2x+3)e^x + (x^2+3x+3)e^x$

$= (x^2+5x+6)e^x$

c) $\lim_{x\to\infty} (x^2+x+2)e^x = \infty \cdot e^{\infty} = \infty$

$\lim_{x\to-\infty} (x^2+x+2)e^x = \lim_{x\to-\infty} \frac{x^2+x+2}{e^{-x}}$ $\quad \infty \cdot 0$ $\quad \frac{\to\infty}{\to\infty}$

$\overset{L'H}{=} \lim_{x\to-\infty} \frac{2x+1}{-e^{-x}}$ $\frac{\to-\infty}{\to-\infty}$ $= \lim_{x\to-\infty} \frac{2}{e^{-x}} = \frac{2}{\infty} = 0$

a) $(x^2+3x+3)e^x = 0 \quad (e^x > 0)$

$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 3}}{2} = \frac{-3 \pm \sqrt{-3}}{2}$

no real sol.

$x^2+3x+3 > 0$

Since $e^x > 0$, $f' > 0$ so f is increasing

b) $(x^2+5x+6)e^x = 0 \quad (e^x > 0)$

$x^2+5x+6 = 0$

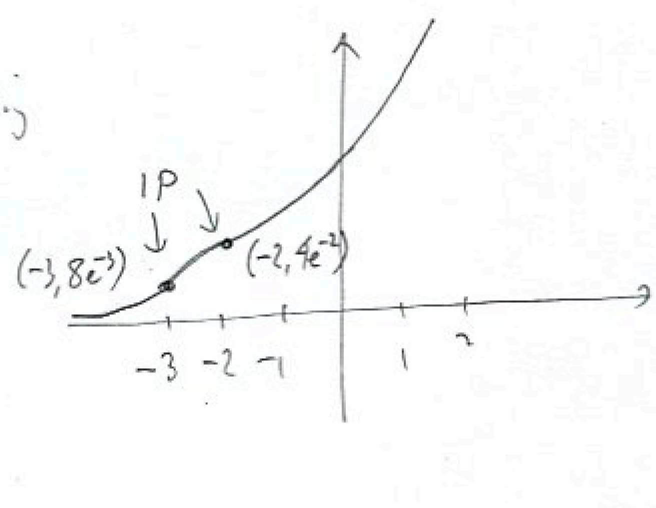
$(x+2)(x+3) = 0$

$x = -2, -3$

	-3		-2	
f''	$+$ 0	$-$	0 $+$	
f	CU IP	CD	IP CU	

d)

x	$(x^2+x+2)e^x$
-2	$4e^{-2}$
-3	$8e^{-3}$



2. (14pts) Let $f(x) = \sin^2 x - \cos x$. Find the absolute minimum and maximum values of f on the interval $[0, \pi]$.

$f'(x) = 2\sin x \cos x - (-\sin x)$

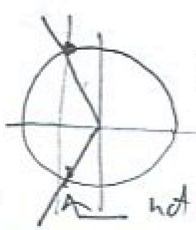
$= \sin x\,(2\cos x + 1)$

$f'(x) = 0$ if

$\sin x = 0$ or $2\cos x + 1 = 0$

$x = 0, \pi$ $\quad \cos x = -\frac{1}{2}$

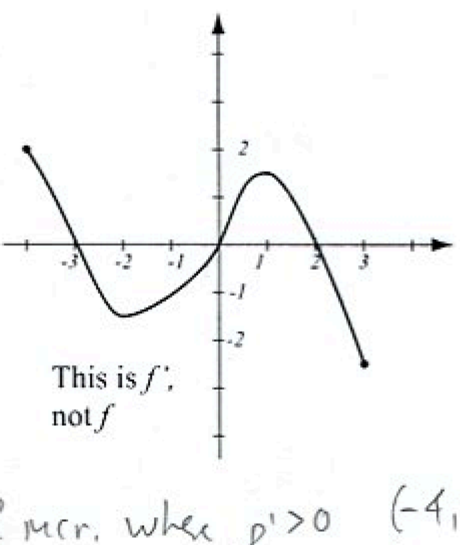
x	$\sin^2 x - \cos x$	
0	$0 - 1 = -1$	min
π	$0 - (-1) = 1$	
$\frac{2\pi}{3}$	$\left(\frac{\sqrt{3}}{2}\right)^2 - \left(-\frac{1}{2}\right) = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}$	max

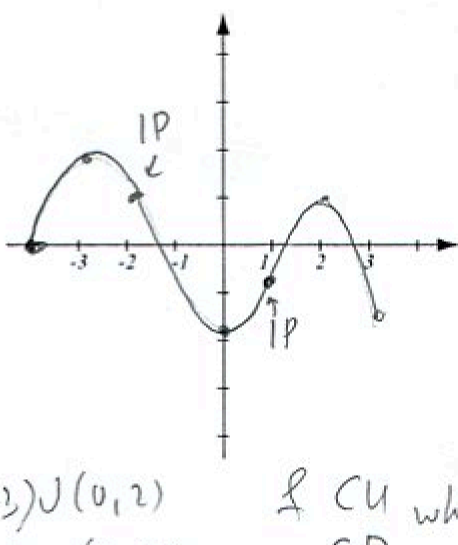


$x = \frac{2\pi}{3}$

not in $[0, \pi]$

3. (18pts) Let f be continuous on $[-4, 3]$. The graph of its derivative f' is drawn below. Use the graph to answer (sign charts may help):
a) What are the intervals of increase and decrease of f? Where does f have a local minimum or maximum?
b) What are the intervals of concavity of f? Where does f have inflection points?
c) Use the information gathered in a) and b) to sketch the graph of f at right, if $f(-4) = 0$.





f incr. when $f' > 0$ $\quad (-4,-3) \cup (0,2)$
decr $\quad < 0$ $\quad (-3,0) \cup (2,3)$

local min at $x = 0$,
max at $x = -3, 2$

f CU when f' incr: $(-2, 1)$
CD $\quad$ decr.: $(-4,-2) \cup (1,3)$

Inflection points at $x = -2, 1$

4. (16pts) Consider $f(x) = x^2 - 3x + 5$ on the interval $[1, 4]$.
a) Verify that the function satisfies the assumptions of the Mean Value Theorem.
b) Find all numbers c that satisfy the conclusion of the Mean Value Theorem.

a) f is a polynomial, hence continuous and diff. on $\mathbb{R}$, and especially on $[1,4]$ and $(1,4)$.

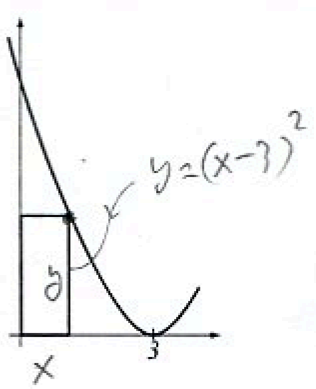
b) $\frac{f(4)-f(1)}{4-1} = \frac{16-12+5-(1-3+5)}{3} = \frac{9-3}{3} = 2$

the solution is in $(1,4)$

$f'(x) = 2x-3$ $\quad$ $2x-3=2$ $\quad$ $x = \frac{5}{2}$ ✓

$2x = 5$

5. (22pts) Consider a rectangle with sides on the x- and y-axes whose one vertex lies on the parabola $y = (x-3)^2$ and is enclosed in the region between the axes and the parabola. Among all such rectangles, find the one with the biggest area.



$A = xy = x(x-3)^2$

Job: maximize $A(x) = x(x-3)^2$ on $[0,3]$

$A'(x) = 1\cdot(x-3)^2 + x\cdot 2(x-3)$

$= (x-3)(x-3+2x)$

$= (x-3)(3x-3)$

$= 3(x-3)(x-1)$

Critical points: $x = 1, 3$

x	$x(x-3)^2$
0	0
3	0
1	4 abs max

Highest $A(x)$ for $x = 1$

Bonus. (10pts) Draw a function, if possible, that satisfies the given conditions. Justify if such a function is not possible.

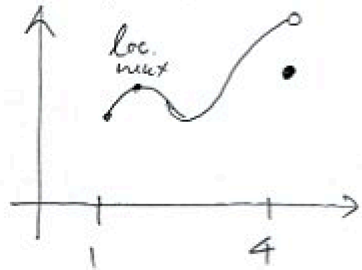
a) f defined on $[1,4]$, has a local maximum but no absolute maximum.

b) f continuous on $[1,4]$, has a local minimum but no absolute maximum.

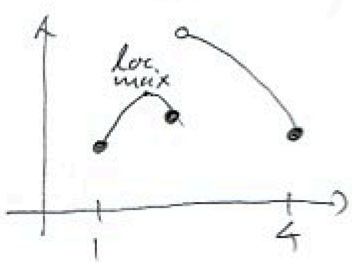
c) f defined on $[1,4)$, has no local minimum nor maximum, and has no absolute minimum nor maximum.

some possibilities

a)



or



b) This is not possible, since a closed function defined on a closed interval always attains its abs. min. and abs. max.

some possibilities

c)